DAPC 2023 Training Sessions Session 3

Verwoerd September 18, 2023

Session 3

- Team Reference Document
- Solutions to Sorting and Search Problems
- Solving interactive problems
- Solving Dynamic Programming Problems

Slides are available on https://chipcie.wisv.ch/ in the training news post.

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Team Reference Document

- During the DAPC you may bring any analogue reference material you want
- Starting from the BAPC, you may only bring limited reference material (= same rules as World Finals)
- This reference is called the Team Reference Document (TRD)
- Sometimes the old term Team Contest Reference (TCR) is used

Each contestant may bring an (identical) copy of a Team Reference Document. This document may contain up to 25 pages of reference materials, single-sided, letter or A4 size, with pages numbered in the upper right-hand corner and your university name printed in the upper lefthand corner. Text and illustrations must be readable by a person with correctable evesight without magnification from a distance of 1/2 meter. It may include handwritten comments and corrections on the fronts of pages only. The document should be in some type of notebook or folder with the name of your institution on the front.

ŤUDelft

Formulas for Geometric Shapes

```
oppervlakte cirkel : \pi r^2
omtrak cirkel : \pi d
oppervlakte ellips : \pi ab
oppervlakte kegel : \pi r^2 + \pi r \sqrt{r^2 h^2}
inhoud kegel : \frac{1}{3} \pi r^2 h
oppervlakte bi : 4\pi r^2
inhoud bol : \frac{4}{3} \pi r^3
oppervlakte cillinder : 2\pi r h + 2\pi r^2
```

More Formulas

1.

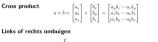
$$\begin{array}{l} \text{sast common multiple}: |\text{cm}(m,n) = \frac{|m\cdot n|}{(m\cdot 1)|m} \\ \text{Catalan number}: C_n = \frac{1}{n+1} \binom{|2n|}{(n+1)|m|} = \frac{2|n|!}{(n+1)|m|} = \prod_{k=2}^n \frac{n+k}{k} \\ \text{Catalan numbers}: C = \{1, 1, 2, 5, 14, 42, 132, 429, 140, 482, 16706\} \\ \text{Triangle numbers}: T = \{1, 3, 6, 10, 15, 21, 28, 36, 45, 55, 66, 78, 91, 105, 120, 136\} \\ \text{Triangle numbers}: T_n = \sum_{n=1}^n k = \frac{m(n+1)}{2} = \binom{n+1}{2} \\ \end{array}$$

Fibonacci Numbers

1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, 377, 610, 987, 1597, 2584

- When we take a pairs of large consecutive Fibonacci numbers, we can approximate the golden ratio by dividing them.
- . The sum of any ten consecutive Fibonacci numbers is divisible by 11.
- · Two consecutive Fibonacci numbers are co-prime.
- The Fibonacci numbers in the composite-number (i.e. non-prime) positions are also composite numbers.

Computational Geometry





Punt in concaaf/convex polygon test

Tel het aantal doorsnijdingen van polygon met lijn P naar oneindig. Als het aantal doorsnijdingen oneven is, dan $P\in ABCDE.$

$$\begin{split} \alpha &= \angle APB + \ldots + \angle DPE + \angle EPA \\ \alpha &= 0 \Rightarrow P \notin ABCDE \\ \alpha &= 2\pi \Rightarrow P \in ABCDE \end{split}$$

Centroid of polygon

The centroid or geometric center of a plane figure is the arithmetic mean ("average") position of all the points in the shape. Informally, it is the point at which an infinitesimally thin cutout of the shape could be perfectly balanced on the tip of a pin.

P

$$\begin{split} C_x &= \frac{1}{6A} \sum_{i=0} (x_i + x_{i+1}) (x_i y_{i+1} - x_{i+1} y_i) \\ C_y &= \frac{1}{6A} \sum_{i=0}^{n-1} (y_i + y_{i+1}) (x_i y_{i+1} - x_{i+1} y_i) \\ A &= \frac{1}{2} \sum_{i=1}^{n-1} (x_i y_{i+1} - x_{i+1} y_i) \end{split}$$





Math

```
int ceildiv(int a, int b) {
   return (a + b - 1) / b;
```

Euler Totient Function (aantal coprimes $\leq n$)

```
Discrete logaritme a^x \equiv b \pmod{m}, retourneert de kleinste x die hieraan voldoet anders 
-1 (maakt gebruik van egcd).
```

```
public long modLog(long a, long b, long m) {
   if (b % execd(a, n)[2] != 0) return -1;
    if (n == 0) return 0:
    long n = (long)sgrt(n) + 1;
    Man (Long) Long) man = new HashMan (Long) ();
    long an = 1:
    for (long j = 0; j < n; j++) {
       if (!map.containsKey(an)) map.put(an, i);
        an - an - a 7 -
    long ain = 1, res = Long. NAX_VALUE:
   for (long i = 0: i < n: i++) {
        long[] is = congruence(ain, b, m);
        for (long aj = is[0]; aj < m; aj += is[i]) {
            if (man.containsKev(ai)) {
                long j = map.get(aj);
                res = min(res, i * n + i);
        if (res < Long. MAX_VALUE) return res:
        ain = ain * an % n;
    return -1:
```

Rekent $(a^b) \mod c$ uit:

int modpow(int a, int b, int c){	
long x=1,y=a; // long is taken to avoid overflow of intermediate	results
while (b > 0) (
if(b%2 == 1){	
x=(x*y)%c;	
}	
<pre>y = (y*y)%c; // squaring the base</pre>	
b /= 2;	
)	
return x%c;	
)	

Rekent $(a \cdot b) \mod c$ uit:

```
lage muleed(lage x, lage b, lage c)(
    lage x o y = x X u;
    vois y = y X u;
    vois y = y x u;
    vois y = y u;
```

Aantal mogelijke manieren om een nummer te splitsen in positieve getallen. Bijvoorbeeld: $f(4) = \{4, 3 + 1, 2 + 2, 2 + 1 + 1, 1 + 1 + 1 + 1\}$.

```
ist partition(ist a) {
    in( jap are ist(a + j);
    dp(0) = 1;
    dp(0) = 1;
    for (a + j);
    for (a + j) = 1; i + c + a; i + 1;
    for (a + j) = 1; i + c + a; i + ( (a + j + j - j) / 2) + r;
    if (a + (a + j + i ) - (a + j + j - j) / 2) + r;
        j (a + (a + j + i ) + (a + j + j + j) / 2] + r;
        j
        p(1) = * dp(1 - (a + j + j + j) / 2] + r;
        j
        return dp(a);
    }
```

Team Integer

1 Contest

2 Mathematics 3 Data structures 4 Numerical 5 Number theory 6 Combinatorial 7 Graph 8 Geometry 9 Strings 10 Various Contest (1) template.cpp finclude (bits/stdc++,b) wing namespace stdu #define rep(i, a, b) for(int i = a) i < (b)) ++i) idefine all(x) begin(x), end(x) #define sz(x) (int) (x).size() typedef long long 11: typedef pair (int, int> pil) typeder pairsint, inty j

int main() {
 cin.tie(0)->sync_with_stdic(0);
 cin.exceptions(cin.failbit);

.bashrc

alias c='g++ -Nall -Nconversion -Wfatal-errors -g -std=c++14 \ -fsanitize=undefined,address' woodsap = 'clear lock' == 'keyrode 66=less greater' #cgps = <>

.vimrc

set cin aw ai is tawd sw+4 ta+50 nu noeb bp-dark ru cul y_0 ni im jk (sec) i im kj (sec) i no j i "Select region and then type illash to hash your selection. Useful for verifying that there aren't mistypes. Ca Hash w topp -dD-P -fpreprocessed \| tr -d '[ispace]' \ | dSum \| cut -c-6

hash.sh

Hashes a file, ignoring all whitespace and comments. Use for # verifying that code was correctly typed. cpp -dD - fpreprocessed | tr -d'(space:]'| mdSaum |cut -c-6

troubleshoot.txt 52 lines	
Pre-submit: Nrite a few simple test cases if sample is not enough. Are time limits close? If so, generate max cases.	
Is the memory usage fine? Could anything overflow? Make sure to submit the right file.	
Wrong answer: Print your solution! Print debug output, as well.	In general, x_i is given
Can your algorithm handle the whole range of input? Read the full problem statement again.	where A'_i is
Have you understood the problem correctly? Any uninitialized variables?	2.2 Re
Confusing N and N, i and j, etc.7 Are you sure your algorithm works7	If $a_n = c_1 c_2$
Are you sure the STL functions you use work as you think? Add some assertions, maybe resubmit.	$x^{k} + c_{1}x^{k-1}$
Co through the algorithm for a simple case. Co through this list again. Explain your algorithm to a teammate. Ask the teammate to look at your code. Co for a small walk, e.g. to the toilet. Is your output format correct? (including whitespace)	Non-distin $a_n = (d_1 n)$ 2.3 Tr
nearies your solution from the start of let a taxameter on it. Intribution error: How your solid or content stars in Coll/J? How your solid or writing motion that the scope of any vector? Any parallel division by 07 food 0 for asseption Any parallel division by 07 food 0 for asseption Doby with remaining (e.g. remapped signals, ase Variou).	
The link exceeded: to you have any possible infinite inpart the your optimize any possible infinite inpart the your optimize in of unrecessary unlat (Deferences) the inpart is the inpart of optimized (Constant excent) but is your testimates this about your algorithm (Must is your testimates this about your algorithm (Must is to max amount of meany your algorithm (Mus	() where V, H
Mathematics (2)	where $r =$
2.1 Equations	2.4 G 2.4.1 T
	The second secon

$$ax^2 + bx + c = 0 \Rightarrow x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

The extremum is given by x = -b/2a.

$$ax + by = e$$

 $cx + dy = f$
 \Rightarrow
 $x = \frac{cd - bf}{ad - bc}$
 $y = \frac{af - ec}{ad - bc}$

In general, given an equation Ax = b, the solution to a variable x_i is given by

$$x_i = \frac{\det A_i}{\det A}$$

where A'_i is A with the *i*'th column replaced by b

2.2 Recurrences

```
If a_n = c_1 a_{n-1} + \cdots + c_k a_{n-k}, and r_1, \ldots, r_k are distinct roots of x^k + c_1 x^{k-1} + \cdots + c_k, there are d_1, \ldots, d_k s.t.
```

```
a_n = d_1 r_1^n + \dots + d_k r_k^n.
```

Non-distinct roots r become polynomial factors, e.g. $a_n = (d_1n + d_2)r^n$.

2.3 Trigonometry

sin(v + w) = sin v cos w + cos v sin wcos(v + w) = cos v cos w - sin v sin w

```
\tan(v + w) = \frac{\tan v + \tan w}{1 - \tan v \tan w}

\sin v + \sin w = 2 \sin \frac{v + w}{2} \cos \frac{v - w}{2}

\cos v + \cos w = 2 \cos \frac{v + w}{2} \cos \frac{v - w}{2}

(V + W) \tan(v - w)/2 = (V - W) \tan(v + w)/2
```

 $(V + W) \tan(v - w)/2 = (V - W) \tan(v + w)/2$ ere V. W are lengths of sides opposite angles v. w.

> $a \cos x + b \sin x = r \cos(x - \phi)$ $a \sin x + b \cos x = r \sin(x + \phi)$

where $r = \sqrt{a^2 + b^2}$, $\phi = \operatorname{atan2}(b, a)$.

2.4 Geometry

```
2.4.1 Triangles
```

```
Side lengths: a, b, c

Semiperimeter: p = \frac{a+b+c}{2}

Area: A = \sqrt{p(p-a)(p-b)(p-c)}

Circumradius: R = \frac{abc}{t,s}
```

KTH

6.2.2 Lucas' Theorem

Let n, m be non-negative integers and p a prime. Write $n = n_k p^k + ... + n_1 p + n_0$ and $m = m_k p^k + ... + m_1 p + m_0$. Then $\binom{m}{m} \equiv \prod_{i=0}^{k} \binom{m_i}{m_i} \pmod{p}$.

6.2.3 Binomials

6.3 General purpose numbers

6.3.1 Bernoulli numbers

EGF of Bernoulli numbers is $B(t) = \frac{t}{t^t-1}$ (FFT-able). $B[0, \ldots] = [1, -\frac{1}{2}, \frac{1}{6}, 0, -\frac{1}{30}, 0, \frac{1}{42}, \ldots]$

Sums of powers:

$$\sum_{i=1}^{n} n^{m} = \frac{1}{m+1} \sum_{k=0}^{m} {m+1 \choose k} B_{k} \cdot (n+1)^{m+1-k}$$

Euler-Maclaurin formula for infinite sums:

$$\begin{split} &\sum_{i=m}^{\infty} f(i) = \int_{m}^{\infty} f(x) dx - \sum_{k=1}^{\infty} \frac{B_k}{k!} f^{(k-1)}(m) \\ &\approx \int_{m}^{\infty} f(x) dx + \frac{f(m)}{2} - \frac{f'(m)}{12} + \frac{f'''(m)}{720} + O(f^{(5)}(m)) \end{split}$$

6.3.2 Stirling numbers of the first kind

Number of permutations on n items with k cycles.

```
c(n,k)=c(n-1,k-1)+(n-1)c(n-1,k),\ c(0,0)=1
```

$$\sum_{k=0}^{n} c(n, k) x^{k} = x(x+1) \dots (x+n-1)$$

 $\begin{array}{l} c(8,k)=8,0,5040,13068,13132,6769,1960,322,28,1\\ c(n,2)=0,0,1,3,11,50,274,1764,13068,109584,\ldots \end{array}$

6.3.3 Eulerian numbers

Number of permutations $\pi \in S_n$ in which exactly k elements are greater than the previous element. k j:s s.t. $\pi(j) > \pi(j + 1)$, k + 1 j:s s.t. $\pi(j) \ge j$, k j:s s.t. $\pi(j) > j$.

$$E(n, k) = (n - k)E(n - 1, k - 1) + (k + 1)E(n - 1, k)$$

 $E(n, 0) = E(n, n - 1) = 1$
 $E(n, k) = \sum_{i=0}^{k} (-1)^{i} \binom{n + 1}{j} (k + 1 - j)^{n}$

multinomial BellmanFord FloydWarshall TopoSort

6.3.4 Stirling numbers of the second kind

```
Partitions of n distinct elements into exactly k groups.
```

S(n, k) = S(n - 1, k - 1) + kS(n - 1, k)

$$S(n, 1) = S(n, n) = 1$$

 $(n, k) = \frac{1}{k!} \sum_{k=1}^{k} (-1)^{k-j} {k \choose j}.$

Total number of partitions of n distinct elements. B(n) = 1, 1, 2, 5, 15, 52, 203, 877, 4140, 21147, For <math>p prime,

 $B(p^m + n) \equiv mB(n) + B(n + 1) \pmod{p}$

6.3.6 Labeled unrooted trees

on n vertices: n^{n-2} # on k existing trees of size n_i : $n_1n_2\cdots n_kn^{k-2}$ # with degrees d_i : $(n-2)!/((d_1-1)!\cdots (d_n-1)!)$

6.3.7 Catalan numbers

$$C_n = \frac{1}{n+1} {\binom{2n}{n}} = {\binom{2n}{n}} - {\binom{2n}{n+1}} = \frac{(2n)!}{(n+1)!n}$$

 $C_0 = 1, \ C_{n+1} = \frac{2(2n+1)!}{n+1!n!} C_n, \ C_{n+1} = \sum C_l C_{n-1}$

 $C_n = 1, 1, 2, 5, 14, 42, 132, 429, 1430, 4862, 16796, 58786, ...$

- sub-diagonal monotone paths in an $n \times n$ grid.
- strings with n pairs of parenthesis, correctly nested.
- binary trees with with n + 1 leaves (0 or 2 children).
- ordered trees with n+1 vertices.
- ways a convex polygon with n + 2 sides can be cut into triangles by connecting vertices with straight lines.
- permutations of [n] with no 3-term increasing subseq.

$\underline{\text{Graph}}$ (7)

7.1 Fundamentals

BellmanFord.h

Description: Calculates shortest paths from s in a graph that might have negative edge weights. Unreachable nodes get dist = inf; nodes reachable through negative-weight cycles get dist = -inf. Assumes $V^2 \max |w_i| < -2^{G1}$. Time: O(VE) = 800 states a state of the state of

```
const 11 inf = LLONG_MAX;
struct Ed { int a, b, w, s() { return a < b ? a : -a; ));
struct Node { 11 dist = inf; int prev = -1; };
```

void bellmanFord(vector<Node>s nodes, vector<Ed>s eds, int s) {

```
nodes[s].dist = 0;
  sort(all(eds), [](Ed a, Ed b) ( return a.s() < b.s()) ));
  int lim = sz(nodes) / 2 + 2; // /3+100 with shuffled vertices
  rep(1,0,1im) for (Ed ed ; eds) (
    Node cur = nodes[ed.a], &dest = nodes[ed.b];
    if (abs/cur.dist) == inf) continue:
    if (d < dest.dist) {
     dest.prev = ed.al
      dest.dist = (i < lim-1 7 d ; -inf);
  rendi.0.1im) for (Ed.e. 1.eds) (
    if (nodesle.al.dist == -inf)
FlowdWarshall h
Description: Calculates all-nairs shortest nath in a directed graph that
might have negative edge weights. Input is an distance matrix m, where
m[i][i] = iof if i and i are not adjacent. As output, m[i][i] is set to the
shortest distance between i and j, inf if no path, or -inf if the path goes
through a negative-weight cycle.
Time: O(N3
                                                      531245, 12 lines
const 11 inf = 11L << 62;
void floydWarshall(vector<vector<ll>>4 m) {
  int n = sz(m);
  rep(k,0,n) rep(i,0,n) rep(i,0,n)
```

11

```
p(s,y,n) rep(s,y,n) rep(y,y,n)
if (m[i](k] != inf 44 m[k](j] != inf) {
    auto newDist = max(m[i](k] + m[k](j], -inf);
    m[i](j] = min(m[i](j], newDist);
```

rep(k,0,n) if (m[k][k] < 0) rep(i,0,n) rep(j,0,n)
if (m[i][k] != inf 44 m[k][j] != inf) m[i][j] = -inf;</pre>

TopoSort.h

Popolovini Topological sorting. Given is an oriented graph. Output is an ordering of vertices, such that there are edges only from left to right. If there are cycles, the returned list will have size smaller than n – nodes reachable from cycles will not be returned. Timer O(1|t + |LE)

```
vi topoSort(const vector<vi>6 gr) {
```

```
vi indeg(sz(gr)), ret;
for (auto6 li : gr) for (int x : li) indeg[x]++;
```

```
queue(int) q; // use priority.queue for lexic. largest ans.
rep(i,0,sx(qr)) if (indeg(i) == 0) q.push(i);
while ((q.empty()) (
int i = q.front(); // top() for priority queue
```

```
ret.push_back(i);
```

```
for (int x : or[i])
```

```
if (--indeg[x] == 0) q.push(x);
```

Potential subjects in a TRD

- 1. Mathematics
 - Formulas and Theories
 - Trigonometry
- 2. Data Structures
 - Segement Tree, Treap, RMQ
 - HashMap, PriorityQueue
- 3. Numerical Methods
 - Simplex, Integration
 - Linear Problem-solving
- 4. Number Theory
 - Primality, Divisability
- 5. Combinatorial
 - Permutations, Partitions

- 6. Graph
 - Search algorithms
 - Flow algorithms
 - Spanning Tree, Connected Components
- 7. Geometry
 - Line intersection, length
 - Triangles and Circles
 - Polygons
- 8. Strings
- 9. Templates
- 10. Tests and reminders

- Only put stuff in that you know how/when to use
- Ensure that the code is correct and complete
- Add short description, complexity, and hash
- Evaluate document after each contest for improvements
- Several templates available at https://chipcie.wisv.ch/resources

Solutions to the sorting and search problems

- Source BAPC Preliminaries 2022
- Time limit: 2s
- For every username calculate the size of the shortest unique prefix.

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• Observation: $n \cdot l \leq 10^7$, so we are aiming for $\mathcal{O}(n \log n)$

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james is closest to jacob and janos, there is no other username that will increase the prefix

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- If we sort the list, we only have to compare with the username before and after
- Alternatively, build a compressed Trie, and for each leaf, count the distance to the root

```
n,l = [int(i) for i in input().split()]
1
   a = sorted([input() for i in range(n)])
\mathbf{2}
3
   t = 1
   p = 0
4
   for i in range(1,n):
5
     for j in range(l):
6
        if a[i-1][j] != a[i][j]:
7
          t += j+1 + max(j-p,0)
8
          p = i
9
          break
10
    print(t)
11
```

- Source BAPC Preliminaries 2022
- Time limit: 2s
- Given *n* algorithms that only work when their input φ is small enough $(\varphi \leq H)$, can you verify the correctness on sufficient large inputs $(\varphi \geq L)$.

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- Add those algorithms to verified algorithms, then find any unverified where $H_j \leq L_i$

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- In this way you can create a graph between the different algorithms.

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- Use a flood fill by BFS/DFS to count the number of algorithms you can reach.

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- We can verify all algorithms with $\mathbf{L}=\varphi_{\mathbf{0}}$
- Add those algorithms to verified algorithms, then find any unverified where $H_j \leq L_i$
- In this way you can create a graph between the different algorithms.
- Use a flood fill by BFS/DFS to count the number of algorithms you can reach.
- This results in an $\mathcal{O}(n^2)$ algorithm.

```
n. k = map(int, input().split())
1
    algs = {tuple(zip(*(map(int, input().split()) for _ in ".."))) for _ in range(n)}
\mathbf{2}
    stack = [[(0, 0) \text{ for } _ \text{ in } range(k)]]
3
    while stack:
4
       base = stack.pop()
\mathbf{5}
       add = {alg for alg in algs if all(l <= b for (a, b), (l, h) in zip(base, alg))}
6
       stack.extend(add)
7
8
       algs = algs.difference(add)
9
    print(n - len(algs))
```

- Source BAPC Preliminaries 2022
- Time limit: 8s
- Given *n* braille characters by their points, determine how many of them are distinct up to translation.

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• Observation 1: time limit of 8s is due to high input size

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- Observation 2: at most 10^6 dots, so we are looking for $\mathcal{O}(n \log n)$

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- Observation 2: at most 10^6 dots, so we are looking for $\mathcal{O}(n \log n)$
- Per Braille character, sort the dots on x then y

- Observation 1: time limit of 8s is due to high input size
- Observation 2: at most 10^6 dots, so we are looking for $\mathcal{O}(n \log n)$
- Per Braille character, sort the dots on x then y
- Move the first ordered dot to (0,0) by subtracting the first point coordinate from all the dots

 $\forall_{i=1}^{m}(x_{i}', y_{i}') = (x_{i} - x_{1}, y_{i} - y_{1})$

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- Add the transposed characters to a HashMap or Set and count the unique keys
- Resulting a $\mathcal{O}(n \log n)$ or amortized $\mathcal{O}(n)$

n, chars = int(input()), set()

```
2 for _ in range(n):
```

- 3 cc = [list(map(int, input().split())) for _ in range(int(input()))]
- 4 min_x, min_y = (min(xs) for xs in zip(*cc))
- 5 chars.add(tuple(sorted([(x min_x, y min_y) for x, y in cc])))
- 6 print(len(chars))

Knitting Pattern

- Source BAPC Preliminaries 2022
- Time limit: 3s
- Given a knitting pattern and amount of wool it costs for letting the wool strand unused, using the wool in a stitch, and for starting or ending the use of wool. Compute the minimal amount of wool required for every colour of wool.

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- Observation: $|p| \leq 10^6$, so we are aiming for a $\mathcal{O}(|p|\log |p|)$

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 $\min(c_{stop} + c_{start}, gap_{size} \cdot c_{unused})$

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```

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```
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```

- · Calculate the total cost for each colour and sum it together
- The complexity is $\mathcal{O}(|w| \cdot |p|)$, or in a single pass over p with creative bookkeeping

```
a, b, c = map(int, input().split())
1
   w = input()
2
   s = input()
3
   for x in w:
4
   off = 😶
5
  on = <u>10</u>**9
6
   for y in s:
7
     if x == v:
8
       on = min(on, off + c) + b
9
         off = on + c
10
       else:
11
12
        on = on + a
     print(off)
13
```

- Source BAPC 2022
- Time limit: 8s
- Find the optimal kiosk position for a given camping layout.

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• The shortest path for every kiosk position to every other plot can be found by using DFS/BFS

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- You an optimize do to some preprocessing, calculate the distance from every plot to every kiosk position, storing intermediate results
- This optimization results in a $\mathcal{O}(n^2)$ solution

Kiosk Construction

```
from collections import deque
 1
 2
 3
     h, w = map(int, input().split())
     plots = [list(map(int, input(), split())) for in range(h)]
 4
 5
     neighs = [[[min((abs(neigh - dest), abs(neigh - plots[v][x]), xx, yy) for xx, yy, neigh in
                     ((xx, yy, plots[yy][xx]) for xx, yy in ((x - 1, y), (x + 1, y), (x, y - 1), (x, y + 1))
 6
                      if 0 <= xx < w and 0 <= yy < h))[2:]
 7
 8
                 for x in range(w) for y in range(h) for dest in range(1, w * h + 1)
 9
10
11
     def find paths(dest x. dest v. dest):
12
       seen, queue = [[0] * w for in range(h)], deque([(dest x, dest v, 0)])
       while queue:
13
14
         x. v. dist = queue.popleft()
         for xx. vv in ((x - 1, y), (x + 1, y), (x, y - 1), (x, y + 1)):
15
           if 0 \le xx \le w and 0 \le yy \le h and neighs[dest - 1][yy][xx] == (x, y) and not seen[yy][xx]:
16
17
             gueue.append((xx, vv, dist + 1))
             seen[vv][xx] = dist + 1
18
19
       return seen
20
21
22
     paths = [[find paths(x, y, plots[y][x]) for x in range(w)] for y in range(h)]
     best = min((max(paths[dest y][dest x][kiosk y][kiosk x] or 1e9 for dest y in range(h) for dest x in range(w)).
23
24
                 plots[kiosk v][kiosk x]) for kiosk v in range(h) for kiosk x in range(w))
     print("impossible") if best[0] == 1e9 else print(*best[::-1])
25
```

24

Solving interactive problems

What are Interactive Problems?

- Traditional problems give all the input at once, you solve and print all the output at once
- Interactive problems give input, you do work, print output, and you receive new input
- This process continues until you find the final answer
- The problem defines an interaction protocol
- The problem may have an interaction limit
- If an interactive problem may be in the set, an simple interactive problem will be included in the test session

- Search in a finite space
- Explore a maze
- Matching games
- Double interaction problem (very, very rare)
 - Program has 2 modes
 - the first mode, input transforms input to output following certain rules
 - The second mode, the output of mode 1 is given and you have tranform it back to the input of mode 1

Common pitfalls for Interactive problems

- Flush the output after every write
 - Only the output, not the input
 - Not flushing the output results in Time Limit Exceeded
- Verdict of a solution is not deterministic, but the following is guaranteed:
 - · Wrong Answer means you printed something wrong
 - Runtime Error means you returned an 0 error code
 - If both occur, you will get either
- ICPC style contests don't have "Idleness Limit Exceeded", but a total runtime limit.

Interactive problems testing tool

- · Most contests provide a testing tool to test the interaction with a testing tool
- This is usually called testing_tool.py in our region
- The header file tells you how to run run the testing tool, for example
 \$ python3 testing_tool.py -f 1.in python3 ./solution.py
- Pitfall for Java/Kotlin: You should run the testing tool in the directory which contains the compiled class file

• Wrong:

~/\$ python3 testing_tool.py -f 1.in java ./code/ProblemA

• Right:

~/code/\$ python3 testing_tool.py -f 1.in java ProblemA

You are asked to guess a number between 0 and *n*.

This is an interactive problem. Your submission will be run against an interactor, which reads from the standard output of your submission and writes to the standard input of your submission. This interaction needs to follow a specific protocol:

The interactor first sends one line with an integer n (3 $\leq n \leq$ 1000), the upper bound of the guessing game.

You can then send a guess g ($0 \le g \le n$).

The interactor will respond with the strings lower, higher, or correct. This represents is if the number to guess is lower, higher, or correct, respectively. After you have guessed the correct number, you should exit the program.

The interactor is not adaptive, i.e. the secret number is fixed during a round. Using more than 12 guesses will result in a wrong answer.

Sample Input 1	Sample Output 1
1000	
	67
higher	
	967
lower	
	500
correct	

```
low = 0
1
   high = int(input())
2
   state = "initial"
3
   while state != "correct":
4
       mid = (high+low)//2
5
       , state = print(mid, flush=True), input().strip()
6
       if state == "higher":
7
           low = mid+1
8
       if state == "lower":
9
            high = mid -1
10
```

Example Interactive Problem: C++ Solution

```
#include <iostream>
 1
     #include <string>
 2
     using namespace std;
 3
 4
 5
     int main() {
       int high.low;
 6
       string state:
 7
 8
       low = 0:
       cin >> high:
 9
       h ob
10
11
         int mid:
12
         mid = (low+high) / 2:
         cout << mid << endl << flush;</pre>
13
14
         cin >> state:
         if(state.compare("higher") == 0) {
15
            low = mid + 1:
16
         } else if(state.compare("lower") ==0 ){
17
18
            high = mid - 1:
19
          ι
        }while (state.compare("correct") != 0);
20
21
        return \Theta:
22
```

Example Interactive Problem: Java Solution

```
import java.io.*:
 2
 3
     public class InteractiveExample {
       public static void main(String... args) throws IOException {
         var output = new BufferedWriter(new OutputStreamWriter(System.out));
         var input = new BufferedReader(new InputStreamReader(System.in));
         var low = 0;
 7
         var high = Integer.parseInt(input.readLine());
 8
         String state:
 9
         do {
10
11
           var mid = (low + high) >> 1:
12
           output.write(mid + "\n"):
           output.flush();
13
           state = input.readLine():
14
           if (state.equals("higher")) {
15
             low = mid + 1:
16
           } else if (state.equals("lower")) {
17
18
             high = mid - 1:
19
         } while (!state.equals("correct"));
20
21
22
```

Example Interactive Problem: Kotlin Solution

```
fun main() {
1
      val output = System.out.bufferedWriter()
2
      var low = 0
3
      var high = readln().toInt()
4
      var state: String
5
      do {
6
        val mid = (low + high) shr 1
7
        output.write("$mid\n")
8
        output.flush()
9
        state = readln()
10
        when (state) {
11
          "higher" -> low = mid + 1
12
          "lower" -> high = mid - 1
13
        }
14
      } while (state != "correct")
15
16
```

Solving Dynamic Programming Problems

- Dynamic Programming (DP) is a techinque of solving problem by solving an problem by solving it in a recursive simpler sub-problem
- DP requires an overlap to occur, else its considered a Divide and Conquer algorithm

Dynamic Programming: Fibonacci Numbers

- The formula is $\mathcal{F}_i = \mathcal{F}_{i-1} + \mathcal{F}_{i-2}$
- It depends clearly on previous calculations
- It can be solved by \mathcal{F}_1 and \mathcal{F}_2 which are both 1

```
1 def fibonacci(n):
2 if n == 1 or n == 2:
3 return 1
4 else:
5 return fibonacci(n - 1) + fibonacci(n - 2)
```

```
1 fun fibonacci(i: Long): Long = when(i) {
2 1L, 2L -> 1
3 else -> fibonacci(i - 1) + fibonacci(i - 2)
4 }
```

Dynamic Programming: Fibonacci Numbers: Caching

- 1 @lru_cache(None)
- 2 def fibonacci(n):
- 3 **if** n == 1 **or** n == 2:
- 4 return 1

```
5 else:
```

```
6 return fibonacci(n - 1) + fibonacci(n - 2)
```

Consider a weighted graph with a adjacency matrix w



Calculate a matrix giving the shortest path from and to all nodes (All Pair Shortest Path (APSP)).

Dynamic Programming: Using States (APSP)

- Calculating Dijkstra for every node is very inefficient
- Create the sub-problem calculate APSP with a subset of the connections
- Define *k* as the number of nodes to use
- Then do the DP by using the following formula:

$$f(i,j,k) = \begin{cases} w(i,j) & \text{if } k = 0\\ \min(f(i,j,k-1), f(i,k,k-1) + f(k,j,k-1)) & \end{cases}$$

- The shortest path between *i* and *j* with using only the first *k* nodes is the min of:
 - the shortest path when using k-1
 - the shortest path from *i* to *k* plus the shortest path from *k* to *j*
- This is Floyd-Warshall's APSP with a complexity $\mathcal{O}(n^3)$

- Source BAPC Preliminaries 2022
- Time limit: 2s
- Given a list of recipes, print the order the recipes by accessibility (lowest <u>beginner time</u> first).

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Cookbook Composition

• Observation: $n \cdot s \le 2.5 \cdot 10^4$, so we are aiming for a $\mathcal{O}(n \cdot s \log n \cdot s)$

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- The beginner time is trivial to calculate, the sum of the time of all steps
- The expert time can be defined as a DP relation

$$time_s = egin{cases} t & ext{if no dependencies are given} \ time_s = egin{cases} t & ext{i=0} \ t + \displaystyle \max_{steps} time_i \end{cases}$$

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$$time_{s} = \begin{cases} t & \text{if no dependencies are given} \\ t + \max_{steps} time_{i} \end{cases}$$

• This can be calculated in linear time by processing the steps one by one, where the expert time is the time of the last step.

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 $\mathsf{time}_{\mathsf{s}} = \begin{cases} t & \text{if no dependencies are given} \\ t + \max_{steps}^{i=0} time_i \end{cases}$

- This can be calculated in linear time by processing the steps one by one, where the expert time is the time of the last step.
- Then sort the recipes based on $\frac{beginner time}{expert time}$ and print out the result

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- This can be calculated in linear time by processing the steps one by one, where the expert time is the time of the last step.
- Then sort the recipes based on <u>beginner time</u> and print out the result
- This results in a $\mathcal{O}(n \cdot s + n \log n)$ solution

```
n, recipes = int(input()), []
 1
 \mathbf{2}
 3
     for in range(n):
       (recipe, s), end = input().split(), {}
       steps = [(sn, int(t), ds) for sn, t, d, *ds in (input().split() for _ in range(int(s)))]
 5
       for step, t, ds in steps:
 6
         end[step] = t + max((end[d] for d in ds), default=0)
 7
       recipes.append((sum(t for _, t, _ in steps) / max(end.values()), recipe))
 8
 9
10
     print("\n".join(name for _, name in sorted(recipes)))
```

Guest Speaker: Maarten Sijm

Guest speaker

Maarten Sijm

- Head of jury for BAPC since 2022
 - FPC jury member since 2018
 - BAPC jury member since 2020
 - NWERC jury member since 2022
- BAPC/NWERC particpant (best result: 24th)
 - 2016: "Tie Limit Exceeded"
 - 2017: "class RubberDuck extends Throwable {}"
 - 2018: "Ω(^{*}/₂)"
- BSc+MSc Computer Science @ TU Delft
- Second-oldest member of CHipCie



- About a dozen jury members
- Tens of problem ideas
- A few months of time
 - · Meeting every two weeks



- Label problems submitted to Call for Problems
 - How much we like the problem
 - Difficulty rating
 - Categories (math, geometry, graph, ...)
- Select problems that we like best, with spread in difficulty and categories



	A	В	с	D	E	F	G	н	1.00	J	к	L	м
1		A	В	с	D	E	F	G	н	I	J	к	L
2	Function optimising	Algorithm				Efficiency Enhance	Function Optimizin	19					
3	List of Cubes		Brewing	Chemistry	Dangerous Chemi	Explosions?		Gas usage					Liquid Mixing Liquid Processing
- 4	k-Bubble Sort		Bubblebubbleso	rt						Interval Sort		k-Bubble Sort	
5	Cookbook Composition	Accessible Cookb	Book of Recipes Beginner Cookboo	Cookbook Comp	osition	Expert Cookbook							
6	Knitting Patterns							Granny's Knitting				Knitting Patterns	
7	Shortest Unique Prefix	Abbreviated Alia	505	Cutting Short									
8	Showerhead				Drilling Holes								
9	Extended Braille		Braille			Extended Braille							
10	Primel (interactive)						Factoring is Futile	Great Guessing G	kame				
11	Slow Memory	Array Adjustment	Book	Copying						Inked Inscription	15		
12	Testing			Comparing Algorit Correctness	Dimensional Data								
13	Moving blocks	Arranging Boxes	Box Arrangement	Cargo Shipping					Heavy Boxes Heavy lifting				

Problem naming sheet of DAPC 2022



Problem Naming

	Α	в	с	D	E	F	G	н	1.1	J	к	L.	м
1		Α	в	с	D	E	F	G	н	1	J	к	L
2	Shortest Unique Prefix	Abbreviated Alia	505	Cutting Short									
3	k-Bubble Sort		Bubble Bubble S	ort						Interval Sort		k-Bubble Sort	
4	Cookbook Composition		Book of Recipes Beginner Cookboo	Cookbook Comp	osition	Expert Cookbook							
5	Testing			Comparing Algorit Correctness	Dimensional Deb								
6	Extended Braille		Braile		Deaf	Extended Braille							
7	Function optimising	Algorithm			Data manipulation		Function Optimisin Fascinating function Fastestest Funct	n	Horrible code		justifying changes justifying junk justifying optimisa junk code		
8	Primel (interactive)						Factoring is Futile	Guessing Game		Integer inquiries			
9	Moving blocks	Arranging Boxes	Box Arrangement	Cargo Shipping					Heavy Boxes Heavy lifting Heavy Hauling		Junk Justified Jetsam		
10	Slow Memory	Array Adjustment	Book	Copying						Inked Inscription	Jumbling Pages		
11	Showerhead				Drilling Holes		Fancy Showerhea	d	Hole drilling		jets jet propulsion jet stream Jabbing Jets		
12	Knitting Patterns			Crochet				Granny's Knitting				Knitting Patterns	
13	List of Cubes		Brewing	Chemistry	Dangerous Chemi	Explosions?		Gas usage			justified jetsam		Liquid Mixing Liquid Processing Lots of Liquid

Problem naming sheet of DAPC 2022

Problem Implementation

- Problem statement (LaTeX)
- Generating test data (YAML spec + C++/Python scripts)
- Validating input (C++/Python scripts)
- Validating output (C++/Python scripts)
- Submissions in all supported languages
- Solution slides (LaTeX)
- Tooling: github.com/RagnarGrootKoerkamp/BAPCtools

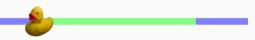
49

- Constraints checking
 - Minimal/maximal input
- Fuzzing
 - Generate more random test data from existing scripts
- Write submissions that should be wrong/too slow
- Invite proof readers/solvers

Check That Everything Works

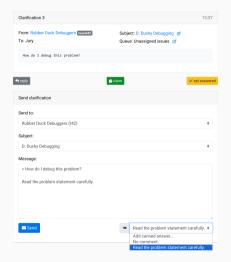
- Continuous Integration
- Upload problems to DOMjudge
 - Local machine
 - Test in Drebbelweg with Maarten (systems Maarten)
- Check time limits on contest hardware

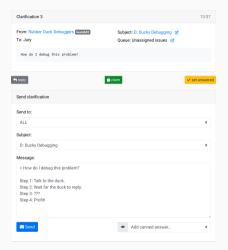
- Waiting for the first submissions to come in
- Taking guesses
 - Which problem would be solved first, and after how many minutes?
 - What will be the order of most-solved to least-solved?



- Check incoming submissions
 - Are they correctly marked as AC/TLE/WA/...?
 - Are they using clever solutions that we didn't think of?
- Answer incoming clarification requests
 - The dreadful "No comment." and "Please read the problem statement carefully."
- Add common mistakes to solution slides

Submissions											
Show	Show: newest unverified unjudged all										
T F	T Filter										
233 st	ubmitted	151 correct	2 unverified								
ID	time	team		problem	lang	result	verified	by	test results		
s233	10:10	team055	'); DROP TABLE teams;	L	кт	CORRECT	no	claim			
s232	09:52	team001	Not The Worst	G	РҮЗ	CORRECT	no	claim			
s231	09:25	team037	Phoenix	Α	JAVA	WRONG-ANSWER	yes	maarten	××××××		
s230	09:25	team042	Rubber Duck Debuggers	Α	CPP	TIMELIMIT	yes	ducky	×××××××		





After the Contest

• Generate solve stats



• Present solutions

And, next year...

• Do it all over again!



Conclusion

Next session is on Thursday the 21st of September.

Guest Speaker: Jeroen op de Beek and Leon van der Waal from Segment goes BRRRR about geometry problems.

https://domjudge.ewi.tudelft.nl/