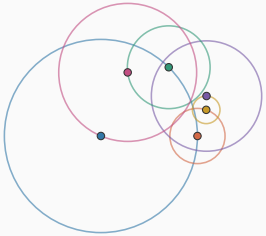
D: Discception

Problem author: Lammert Westerdijk



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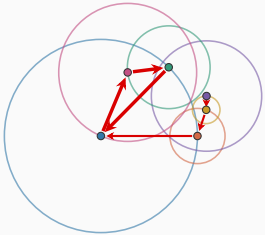
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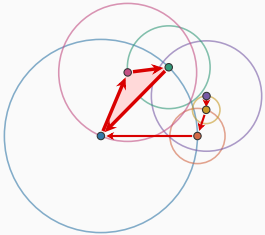
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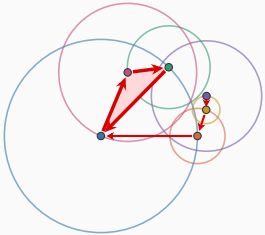


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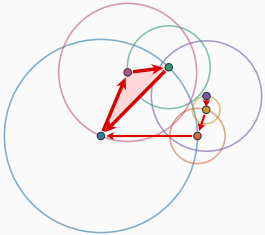
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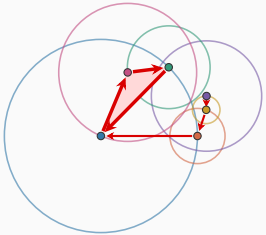


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Goal: Find a subset of r_i that can be the edges of a polygon, this is a necessary and sufficient condition.

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Statistics: 122 submissions, 36 accepted, 14 unknown