# Delft Algorithm Programming Contest (DAPC) 2024

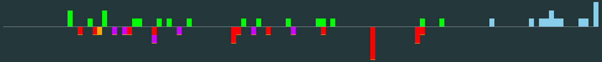
Solutions presentation

The BAPC 2024 jury

September 28, 2024

# A: Awkward Auction

Problem author: Mees de Vries



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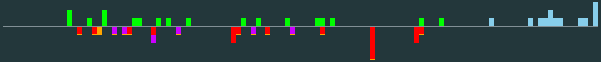

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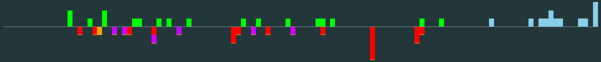

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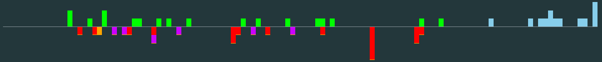

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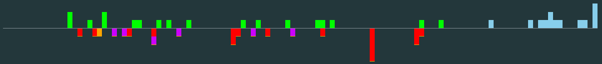

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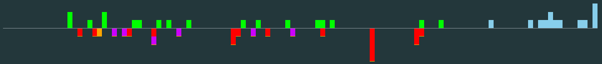

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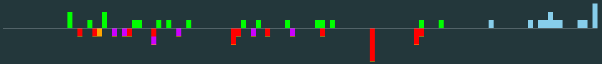

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**Run time:** $\mathcal{O}(n^3)$.

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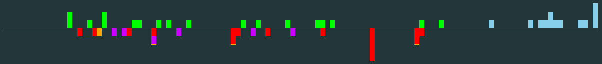

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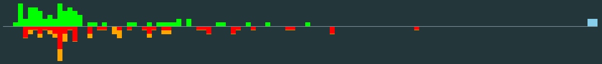
Statistics: 54 submissions, 18 accepted, 13 unknown

# B: Battle of Nieuwpoort

**Problem author: Timon Knigge**



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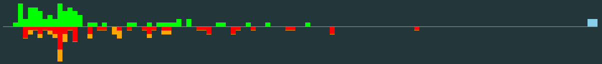

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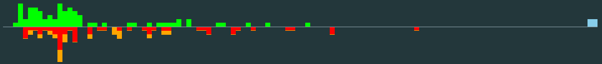

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**Solution (math):** Check if $b^2 \mid y$ using integer modulus:

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y % (b * b) == 0
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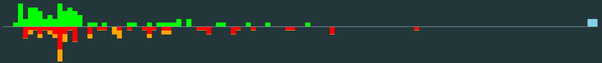

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**Solution (string):** Check if $y$ written in base-$b$ ends with "00". Some programming languages support this natively, such as Java's `Integer.toString(y, b)`. You can also do this digit by digit:

$letters$ = "`0123456789abcdef`"
$s$ = ""
**while** $y > 0$:
    $s$ += $letters[y \ \% \ b]$
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**return** reversed($s$)

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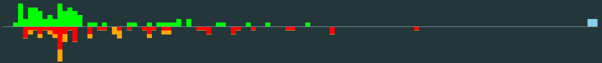

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Statistics: 142 submissions, 71 accepted, 4 unknown

# C: Chaotic Cables

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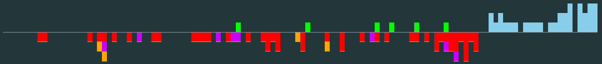

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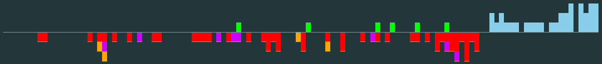

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- Pick an arbitrary vertex $v$ and label it as 0.
- Label all neighbours of $v$ with distinct powers of 2.
- Do a breadth-first search from $v$. For each unvisited neighbour $u$ of $v$, label $u$ with the bitwise OR of its current label and the label of $v$.
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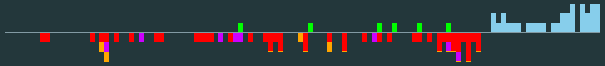

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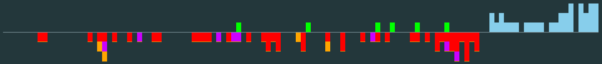

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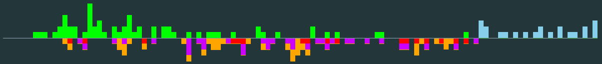
Statistics: 98 submissions, 6 accepted, 32 unknown

# D: Dialling Digits

Problem author: Ragnar Groot Koerkamp



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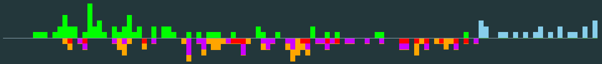

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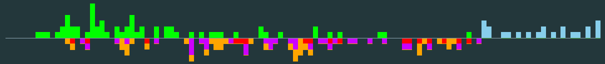

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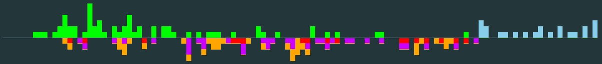

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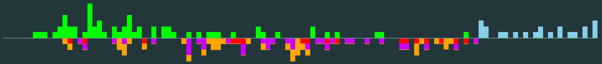

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Statistics: 176 submissions, 63 accepted, 22 unknown

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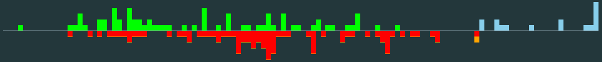

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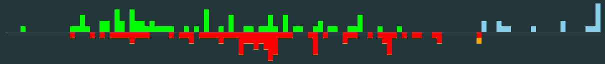

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**Solution:** Let us measure time in deciseconds to avoid decimals.

- If the password is wrong, this adds $4 + n$ deciseconds to your total time.
- We find `continue` yields an expected time of $1 + n - k + (4 + n)p/100$ deciseconds.
- We find `backspace` yields an expected time of $2 + n - k + (4 + n)(1 - p/100)$ deciseconds.
- We find `restart` yields an expected time of $4 + n$ deciseconds.
- To avoid decimals again, compare $100(1 + n - k) + (4 + n)p$ with $100(2 + n - k) + (4 + n)(100 - p)$ and $100(4 + n)$.
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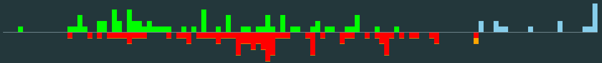

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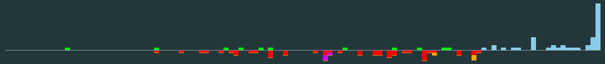
# F: Fractal Area

**Problem author: Lammert Westerdijk**



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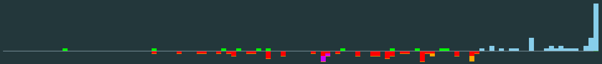
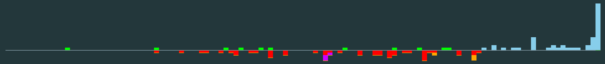


**Problem:** Determine the area of a triangle, where the edges are a fractal defined by a polyline.

**Solution base:** The area of the equilateral triangle with sides of length 1 is $\sqrt{3}/4$.

# F: Fractal Area

**Problem author: Lammert Westerdijk**



**Problem:** Determine the area of a triangle, where the edges are a fractal defined by a polyline.

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**One step:** The area below the polyline can be calculated using the trapezoidal rule:

$$\sum_{i=1}^{n-1} (x_{i+1} - x_i) \cdot \frac{1}{2}(y_i + y_{i+1})$$

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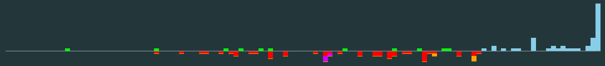

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**Next step:** If the area of level $k$ of the fractal is $A_k$, the area of the next level is multiplied by the square of lengths of the line segments:

$$A_{k+1} = \sum_{i=1}^{n-1} d(i, i+1)^2 A_k \qquad \big(d(i,j) \equiv \text{distance between points } i \text{ and } j\big)$$

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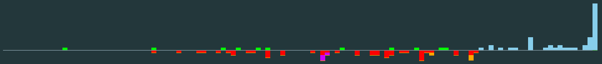

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**Final answer:** Sum areas of all levels and multiply by the 3 sides:

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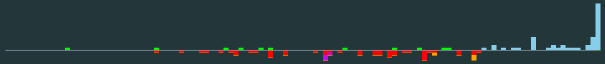

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But summing to $\infty$ is difficult... [*citation needed*]

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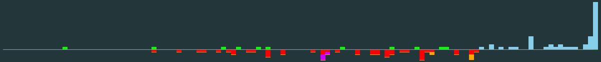

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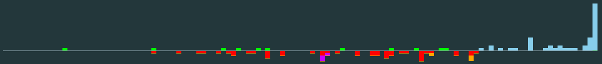

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The sum of a geometric series is $\sum_{k=0}^{\infty} r^k \cdot A_0 = \frac{A_0}{1-r}$.

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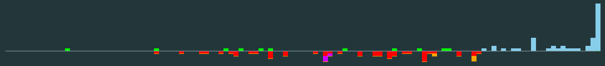

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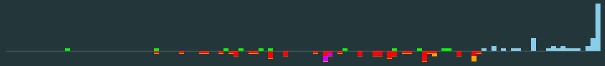

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**Run time:** $\mathcal{O}(n)$ to calculate $A_0$ (area below polyline) and $r$ (sum of squares of segment lengths).

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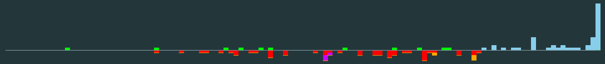

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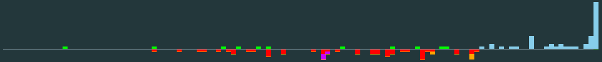

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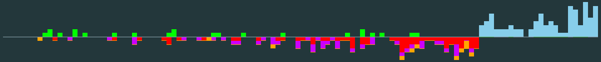
Statistics: 106 submissions, 11 accepted, 45 unknown

# G: Giganotosaurus Game

**Problem author: Tobias Roehr**



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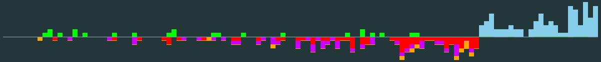

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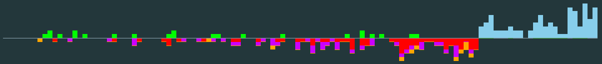

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**Solution:** Let $A[x][k]$ denote the number of paths to cell $x$ with exactly $k$ jumps. You can reach this state by either jumping or not, so

$$A[x][k] = \begin{cases} 0 & \text{if there is a cactus at } x \\ A[x-k-1][k-1] + A[x-1][k] & \text{otherwise} \end{cases}$$

So we use *dynamic programming*. The answer is the sum of all values in $A$ past $n$.

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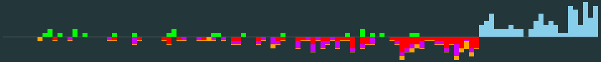

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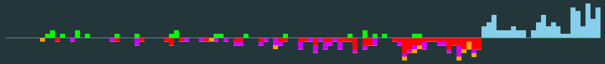

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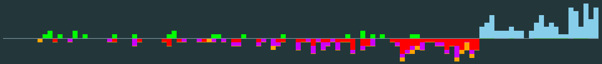

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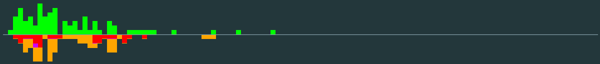
Statistics: 232 submissions, 23 accepted, 90 unknown

# H: Human Pyramid

**Problem author: Mees de Vries**



**Problem:** Find the highest possible pyramid you can build with $n \leq 10^{12}$ people.

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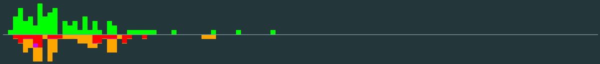

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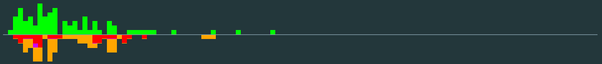

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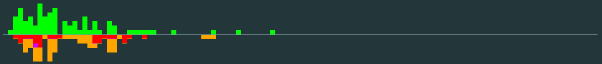

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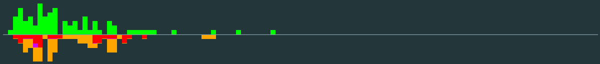

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**Solution 3:** Invert the function $p$: $p^{-1}(n) = \left\lfloor \dfrac{\sqrt{8n+1}-1}{2} \right\rfloor$. $\mathcal{O}(1)$.

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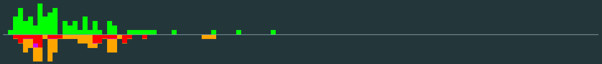

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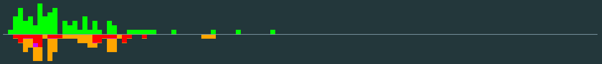

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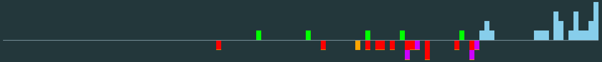
Statistics: 143 submissions, 75 accepted

# I: Investment Investigation

Problem author: Ivan Fefer



**Problem:** Given a list of all orders made on a stock market, generate a list of all transactions made. Normal orders can be fulfilled after being placed, while FoK orders need to be fulfilled instantaneously or not at all.

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**Observation:** Every transaction completes at least one order, so the number of transactions is $\mathcal{O}(n)$.

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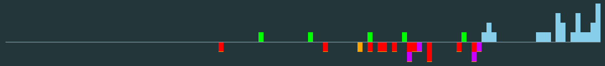

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**Observation:** Normal orders can be handled with a priority queue.

- Run time: $\mathcal{O}(\#\text{transactions} \cdot \log n)$ per order.

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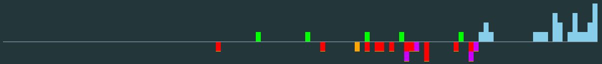

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**Naive solution:** Try handling a FoK order the same as a normal order, undoing transactions if it is not fulfilled.

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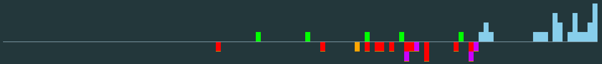

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**Problem:** Can take $\mathcal{O}(n \log n)$ time per FoK order!

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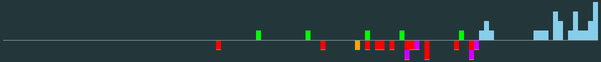

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- Run time: $\mathcal{O}(\#\text{transactions} \cdot \log n)$ per order.

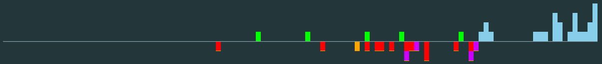
**Naive solution:** Try handling a FoK order the same as a normal order, undoing transactions if it is not fulfilled.

**Problem:** Can take $\mathcal{O}(n \log n)$ time per FoK order!

**Observation:** Need to quickly check whether a FoK order can be fulfilled.

# I: Investment Investigation

Problem author: Ivan Fefer



**Problem:** Given a list of all orders made on a stock market, generate a list of all transactions made. Normal orders can be fulfilled after being placed, while FoK orders need to be fulfilled instantaneously or not at all.

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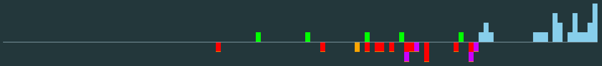

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**Online Solution:** Use augmented binary search tree or implicit segment tree to compute total volume of outstanding orders above a buy price / below a sell price in $\mathcal{O}(\log n)$.

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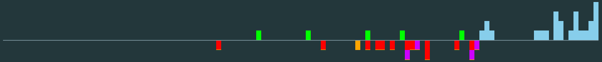

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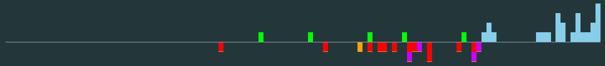

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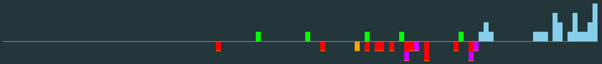

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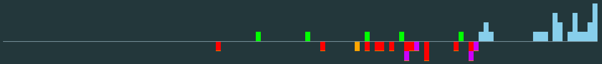

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Statistics: 46 submissions, 5 accepted, 24 unknown

# J: Joppiesaus Jailbreak

Problem author: Mike de Vries

**Problem:** Given the lengths $x_1, \ldots, x_n$ of all levels in a platformer, all of which take an integer number of frames to finish, determine the fastest time to finish all levels if the framerate can be set to any real in $(0, f]$.

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**Reformulating:** With framerate $f'$, each level takes $\lceil x_i f'/1000 \rceil$ frames to finish. The total time is $(1/f') \sum_{i=1}^{n} \lceil x_i f'/1000 \rceil$.

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**Observation 1:** The function $1/f'$ is decreasing, so a minimum can only be attained when $\sum_{i=1}^{n} \lceil x_i f'/1000 \rceil$ jumps, **or when $\mathbf{f'} = \mathbf{f}$**. Jumps occur whenever $f' = 1000m/x_i$ for some integer $0 < m \leq x_i f/1000$.

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**Naive solution:** Compute all interesting framerates, and for each compute the total time to finish the game. This is $\mathcal{O}(nf \sum_{i=1}^{n} x_i/1000)$, too slow!

**Problem:** Given the lengths $x_1, \ldots, x_n$ of all levels in a platformer, all of which take an integer number of frames to finish, determine the fastest time to finish all levels if the framerate can be set to any real in $(0, f]$.

**Observation 2:** If all jumps are distinct, the total number of frames increases by exactly 1 at each jump. If we sort the jumps, recomputing the total time takes $\mathcal{O}(1)$! This also works if the jumps are not distinct.

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**Solution:** Compute all jumps and sort them. For the first jump, compute the total frames. For each jump after the first, simply add a single additional frame. Finally, compute the case $f' = f$.

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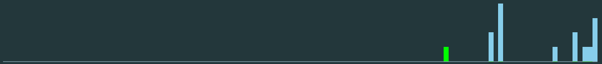
Statistics: 47 submissions, 1 accepted, 39 unknown

# K: Kitchens of Königsberg

**Problem author: Tobias Roehr**



**Problem:** Given multigraph $G$, integer $k$. Find $K \subseteq V(G)$ such that exactly $k$ edges have at least an endpoint in $K$. Also known as “Partial Exact Vertex Cover”.

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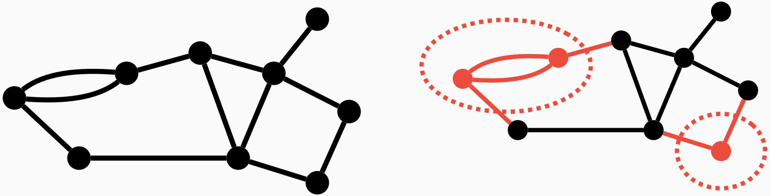
**Example:** for $k = 6$:



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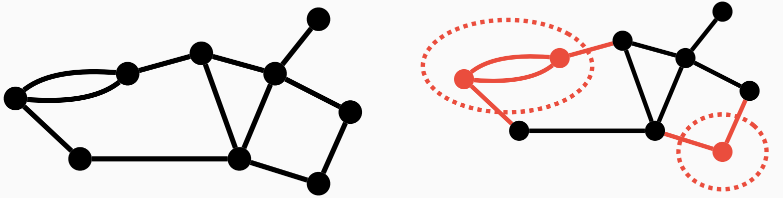

**Naive solution 1:** Consider all $2^n$ subsets of $V(G)$. Running time $\mathcal{O}(2^n \operatorname{poly}(n))$, way too slow.

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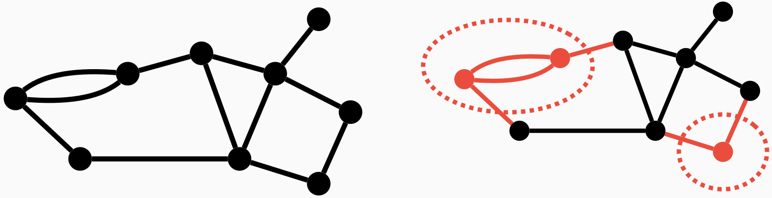

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**Naive solution 2:** Can assume $|K| \leq k$, so it suffices to consider all $\binom{n}{1} + \cdots + \binom{n}{k} \leq n^k$ vertex subsets of size at most $k$. Running time $\mathcal{O}(n^k \operatorname{poly}(n))$, still too slow.

**Problem:** Given multigraph $G$, integer $k$. Find $K \subseteq V(G)$ such that exactly $k$ edges have at least an endpoint in $K$. Also known as "Partial Exact Vertex Cover".

**Hacky solution:** The solution is very small ($|K| \leq k \leq 6$), so we can use preprocessing, exhaustive search, and local optimisation to solve what is otherwise an NP-hard problem even on large instances. Note that it must run in $\mathcal{O}(n^2)$.

Here are some ideas:

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**Remove large degree vertices:** No vertex of degree $> k$ can contribute to the solution (it would cover $> k$ edges), so we can remove those from vertices under consideration for the vertex cover.

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**Check all singletons and pairs:** The vertex set is small enough ($n \leq 5000$) that we can exhaustively check all $K \subseteq V(G)$ with $|K| \leq 2$.

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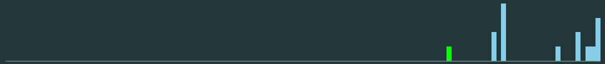

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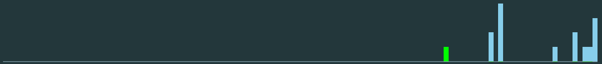

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**Fancy solution:** Random orientation algorithm, see next slide.

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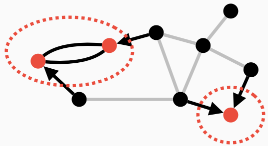
Statistics: 15 submissions, 1 accepted, 14 unknown

**Problem:** Given multigraph $G$, integer $k$. Find $K \subseteq V(G)$ such that exactly $k$ edges have at least an endpoint in $K$. Also known as "Partial Exact Vertex Cover".

*Random orientation algorithm*. Randomly orient each edge $uv$ as either $(u, v)$, $(v, u)$, or leave it undirected, each with probability $\frac{1}{3}$. Compute components $C_1, \ldots, C_r$ such that each $C_i$ only contains arcs pointing *into* $C_i$. (Say, using BFS.) Assemble solution from these $C_i$. ("Subset Sum" the indegrees of components to make $k$.)



*Correctness* Every internal edge in $K$ must remain undirected (probability $\frac{1}{3}$) and every edge incident on $K$ must be directed towards (probability $\frac{1}{3}$). (Orientation of remaining edges unimportant.) Total success probability $= \frac{1}{3}^k$. Do $t = 3^k \ln n$ independent repetitions; all fail with probability

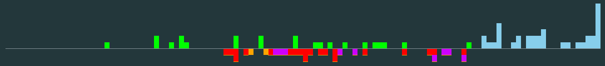
$$\left(1 - \frac{1}{3}^k\right)^t \leq \left(\exp(-\frac{1}{3}^k)\right)^t \leq 1/n\,.$$

Run time $\mathcal{O}(3^k \operatorname{poly}(n))$, known as "fixed parameter tractable (FPT) in $k$".

[Kneis, J., Langer, A., Rossmanith, P. Improved Upper Bounds for Partial Vertex Cover. Graph-Theoretic Concepts in Computer Science. WG 2008. Springer LNCS 5344.]

# L: Lawful Limits

**Problem author: Dirk van Bree**



**Problem:** Find the length of the shortest path through a graph where the maximum speed of all edges increases at some time $t$.

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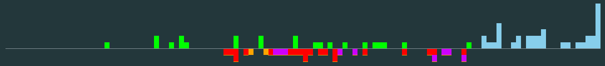

**Problem:** Find the length of the shortest path through a graph where the maximum speed of all edges increases at some time $t$.

**Possible pitfall:** If the speed limit increases when you are on a road, you can drive at that higher velocity instead.

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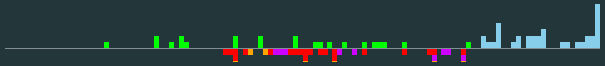

**Problem:** Find the length of the shortest path through a graph where the maximum speed of all edges increases at some time $t$.

**Possible pitfall:** If the speed limit increases when you are on a road, you can drive at that higher velocity instead.

**Remark:** The time it takes to drive down a road of length $\ell$ with speeds $v_1 < v_2$ changing at time $t$ is given by

$$\text{time} = \begin{cases} \ell / v_2 & T \geq t \\ \ell / v_1 & (t - T) \cdot v_1 > \ell \\ t - T + (\ell - (t - T) \cdot v_1) / v_2 & \text{else.} \end{cases}$$

when the current time is $T$.

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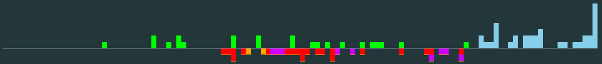

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**Solution:** Apply Dijkstra to the time it takes to get to a vertex.

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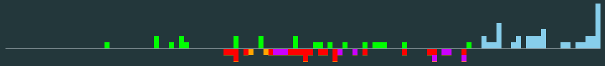

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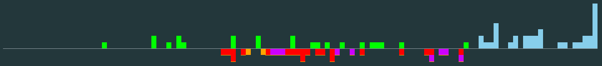
**Solution:** Apply Dijkstra to the time it takes to get to a vertex.

**Run time:** $\mathcal{O}(m + n \log n)$.

# L: Lawful Limits

Problem author: Dirk van Bree



**Problem:** Find the length of the shortest path through a graph where the maximum speed of all edges increases at some time $t$.

**Possible pitfall:** If the speed limit increases when you are on a road, you can drive at that higher velocity instead.

**Remark:** The time it takes to drive down a road of length $\ell$ with speeds $v_1 < v_2$ changing at time $t$ is given by

$$\text{time} = \begin{cases} \ell / v_2 & T \geq t \\ \ell / v_1 & (t - T) \cdot v_1 > \ell \\ t - T + (\ell - (t - T) \cdot v_1) / v_2 & \text{else.} \end{cases}$$

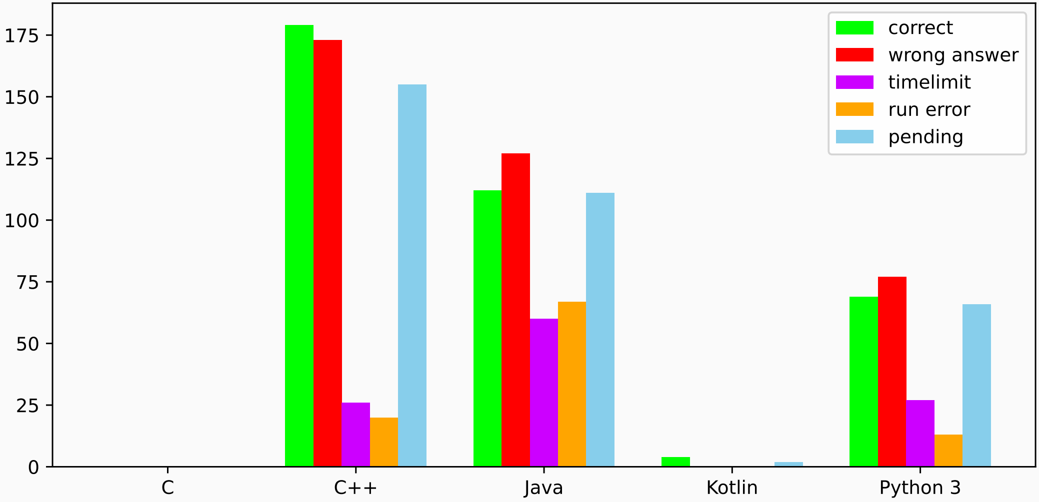
when the current time is $T$.

**Solution:** Apply Dijkstra to the time it takes to get to a vertex.

**Run time:** $\mathcal{O}(m + n \log n)$.

Statistics: 90 submissions, 23 accepted, 35 unknown

# Language stats



# Random facts

## Jury work

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- 505 commits (last year: 492)
- 1228 secret test cases (last year: 1050) ($\approx 102\frac{1}{3}$ per problem!)
- 236 jury + proofreader solutions (last year: 195)
- The minimum[1] number of lines the jury needed to solve all problems is

$$4 + 3 + 7 + 3 + 2 + 3 + 21 + 1 + 60 + 21 + 61 + 9 = 195$$

  On average $16\frac{1}{4}$ lines per problem, up from 13.9 in last year's preliminaries

[1] With some code golfing

# Thanks to:

## The proofreaders

Angel Karchev
Arnoud van der Leer
Jaap Eldering
Jeroen Bransen (Java Hero)
Kevin Verbeek
Pavel Kunyavskiy (Kotlin Hero)
Thomas Verwoerd (Kotlin Hero)
Wendy Yi

## The jury

Gijs Pennings
Jonas van der Schaaf
Jorke de Vlas
Lammert Westerdijk
Maarten Sijm
Mees de Vries
Mike de Vries
Ragnar Groot Koerkamp
Reinier Schmiermann
Thore Husfeldt
Tobias Roehr
Wietze Koops

Want to join the jury? Submit to the Call for Problems of BAPC 2025 at:

`https://jury.bapc.eu/`