

# **Benelux Algorithm Programming Contest (BAPC) 2024**

Solutions presentation

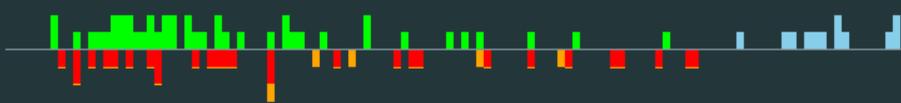
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The BAPC 2024 jury

October 29, 2024

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Problem author: Freek Henstra



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- Look at the lowest ranking team with pending submissions.
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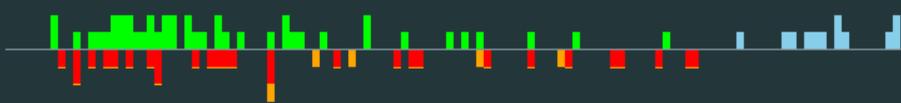
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Statistics: 90 submissions, 44 accepted, 12 unknown

# B: Buggy Blinkers

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**Solution:** Perform a (breadth-first) search on a higher-dimensional space, where each “hypernode” is defined by

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Prune the search if  $\#activations > k$ .

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**Running time:** With  $n \cdot 4 \cdot k \cdot 3$  hypernodes and each node having  $\mathcal{O}(1)$  edges, BFS takes  $\mathcal{O}(kn)$  time. Dijkstra with running time  $\mathcal{O}(kn \log n)$  is accepted, but not necessary.

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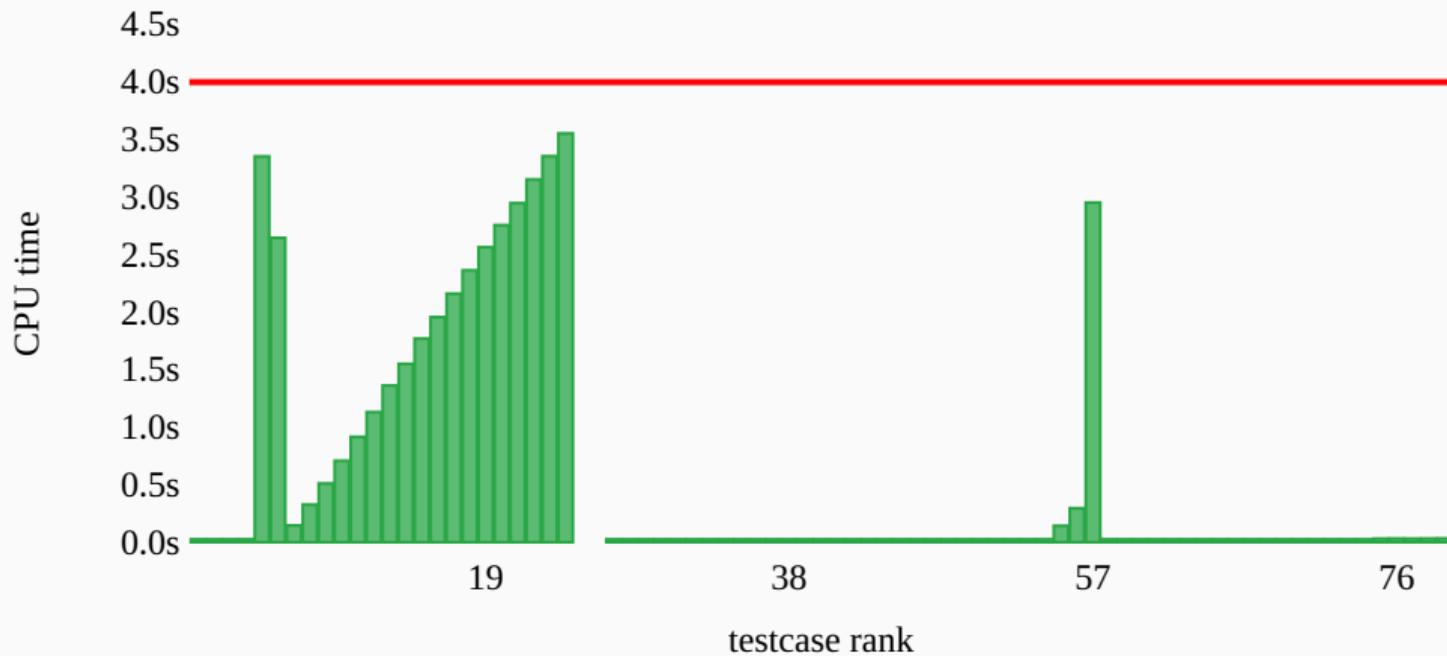
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Statistics: 49 submissions, 9 accepted, 25 unknown

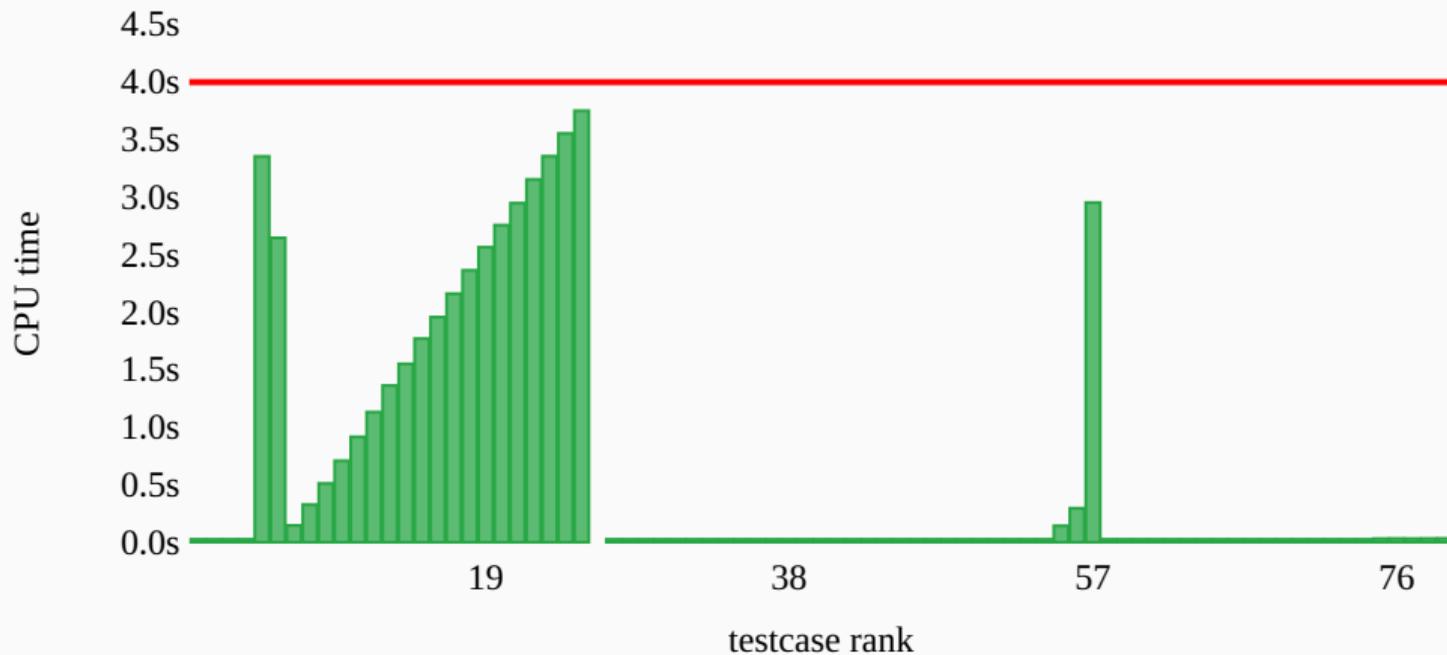
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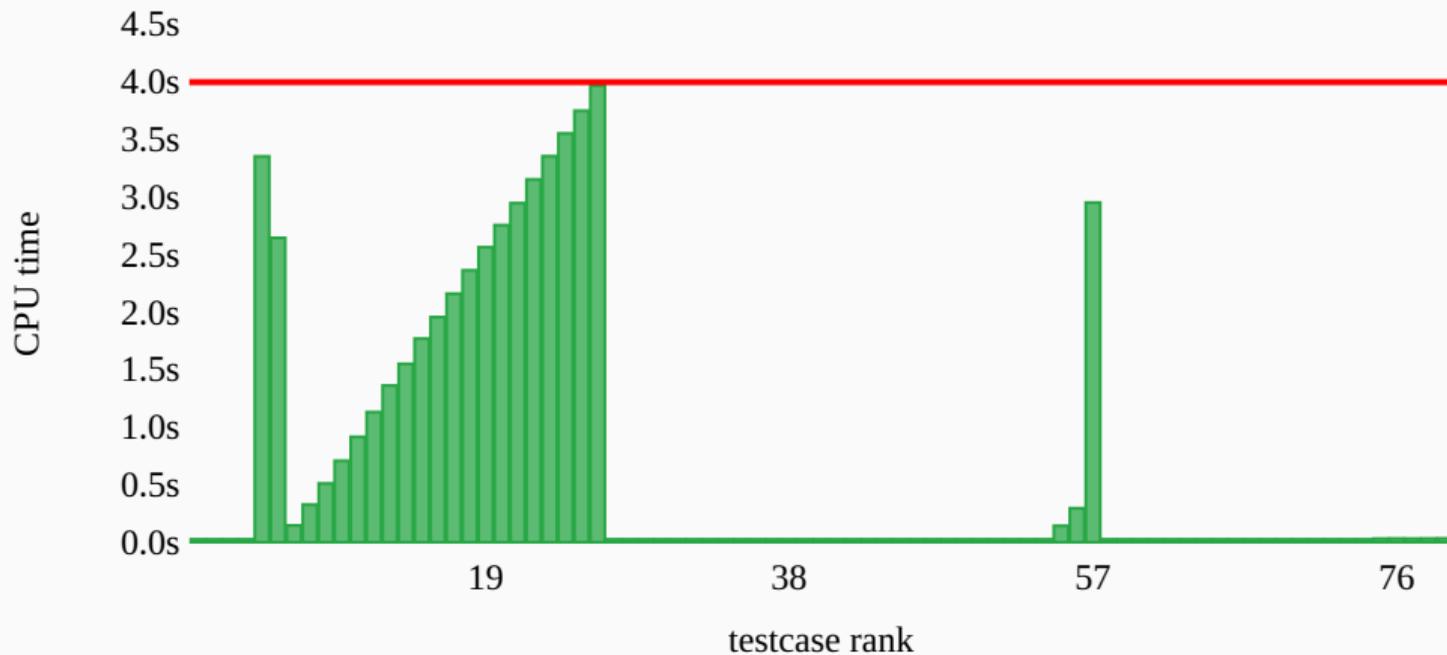
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Only 0.032 seconds to spare!

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**Proof sketch:** Given a solution, if a person wants to switch, everyone with lower skill also wants to. If someone wants to switch at the end, a contestant with higher skill would have picked a different contest. They didn't, so this must be optimal.

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**Pitfall:** Floating point numbers: The expected value of joining a contest is given by

$$\frac{\text{prize} \cdot \text{skill}}{\text{total skill in contest}}$$

Comparing these values naively will lead to floating point errors. Instead, use

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Statistics: 20 submissions, 4 accepted, 13 unknown

# D: Disgruntled Diner

Problem author: Wietze Koops



**Problem:** Given a table number  $t$  and menu item  $m$ , determine which pinned-up tickets must be flipped to prove the following claim:

$$\forall \text{ pinned-up tickets "Ld"} : (d = t) \rightarrow (L = m).$$

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$$\forall \text{ pinned-up tickets "Ld"} : (d = t) \rightarrow (L = m).$$

**Observation:** The claim is false if and only if

$$\exists \text{ pinned-up ticket "Ld"} : (d = t) \wedge (L \neq m).$$

Let's call such tickets *illegal*.

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**Claim:**  $\forall$  pinned-up tickets “Ld” :  $(d = t) \rightarrow (L = m)$ .

**Definition:** A ticket “Ld” is *illegal* if  $(d = t) \wedge (L \neq m)$ .

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- Solution:**
1. *Partition* the computer tickets into sets of legal and illegal tickets. If there are no illegal tickets, we can immediately return “true”.
  2. Compute a *bipartite matching* between the legal tickets and the pinboard. If this is impossible, the claim is guaranteed to be “false”.
  3. Now, a matching exists, but any legal ticket on the board could be replaced by an illegal one if the upright side is the same. So, a ticket “x” must be *flipped* if:
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Statistics: 3 submissions, 0 accepted, 3 unknown

# E: Extraterrestrial Exploration

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**Problem:** Given query access to a sorted list of integers  $a_1, a_2, \dots, a_n$ , determine  $x, y, z$  that maximize

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**Naive solution:** Check all possible triples and compute the maximum. This is  $\mathcal{O}(n^3)$ , which is too slow, but more importantly, there are way too few queries to determine the values of all  $a_i$ !

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**Pitfall:** Make sure to return distinct indices!

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**Testing tool:** There was an issue with the provided testing tool: solutions were compared by computing

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as a floating-point number.

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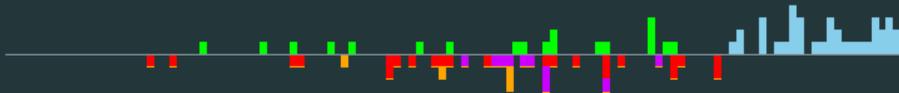
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Statistics: 159 submissions, 34 accepted, 33 unknown

# F: Failing Factory

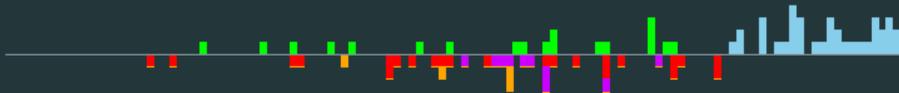
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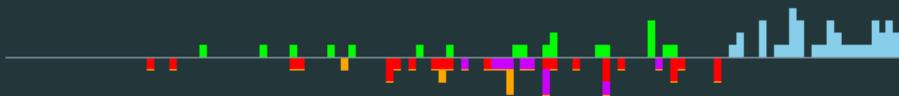


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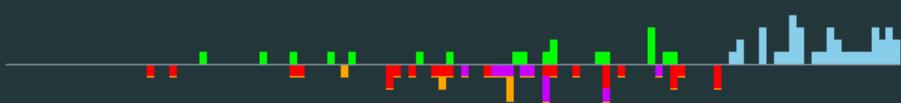
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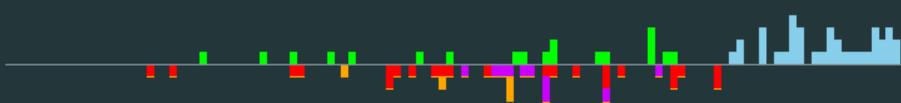
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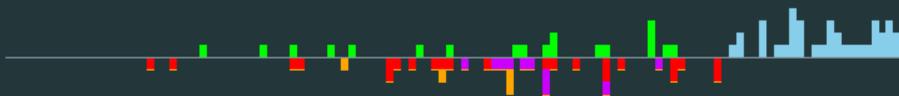
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**Insight 2:** We should look for a SCC without external dependencies. (So a sink in the collapsed graph).

**Solution:** Use Tarjan's or Kosaraju's algorithm to find strongly connected components.

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**Problem:** Given is a graph of the dependencies between steps in a factory. Each step independently fails with some probability  $p_i$ . Find the maximum probability that a step and all its dependencies do not fail.

**Naive solution:** For each step, multiply the success probabilities of all its dependencies, using DFS.  $\mathcal{O}(n^2)$  is too slow!

**Insight 1:** Within a strongly connected component, all steps have the same failure probability.

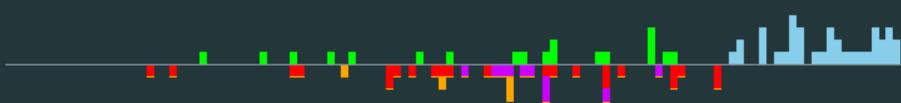
**Insight 2:** We should look for a SCC without external dependencies. (So a sink in the collapsed graph).

**Solution:** Use Tarjan's or Kosaraju's algorithm to find strongly connected components.

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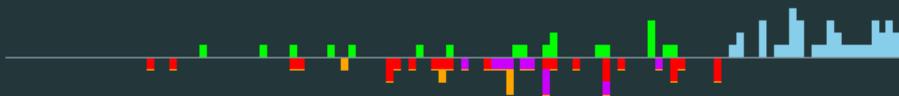
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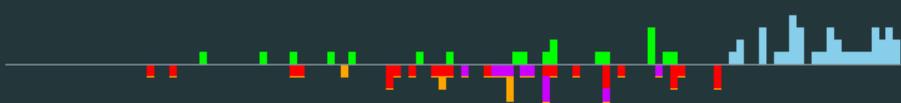
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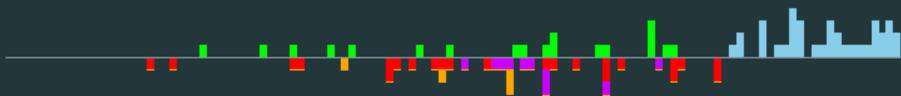
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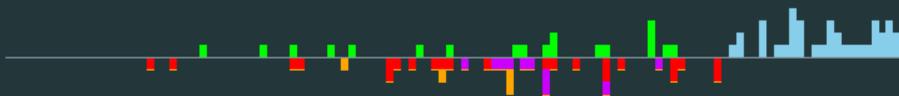
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Statistics: 92 submissions, 19 accepted, 36 unknown

# G: Grocery Greed

Problem author: Jorke de Vlas



**Problem:** Given a list of prices, divide them into groups and decide for each group whether to round the price of the group to a multiple of 5 cents, to minimize the total price.

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**Solution:**

- As long as there is both a price of 3 cents and a price of 4 cents, put them together in a group and round to 5 cents.

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- As long as there is both a price of 3 cents and a price of 4 cents, put them together in a group and round to 5 cents.
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**Pitfall:** Be careful converting between floating point numbers and integers!

- Casting `100*x` (which is `float`) to `int` is flooring, so add `0.5` or use `round()`.
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Statistics: 111 submissions, 44 accepted, 10 unknown

# H: Horse Habitat

Problem author: Mike de Vries



**Problem:** Given a square grid with  $r$  rows and  $c$  columns, each square being either '.' or '#'. Determine for each  $1 \leq w \leq c$  and  $1 \leq h \leq r$  the number of  $w \times h$  rectangles in the grid with only '.'.

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**Running time:** For  $n = rc$ :  $\mathcal{O}(rc^2) = \mathcal{O}(n\sqrt{n})$  after possibly transposing to make sure  $c \leq r$ . Too slow!

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**New plan:** For  $h$  from  $r$  down to 1, we can keep track of which squares  $(i, j)$  have  $d(i, j) \geq h$ . We can then determine the number of  $w \times h$  rectangles for any  $w$  by counting the number of intervals of length  $w$  in each row.

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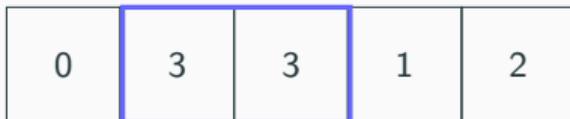


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#....



$h = 3$

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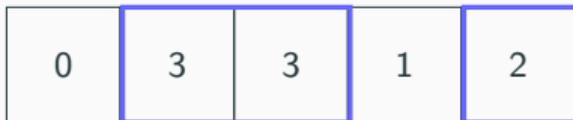


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$h = 2$

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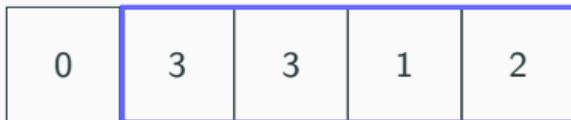


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**Calculation:** After adding all squares with values of at least  $h$ , let  $I_w$  be the number of maximal intervals of length  $w$ . The total number of  $w \times h$  rectangles can then be calculated as  $R_w = I_w + 2I_{w+1} + \dots$

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**Running time:** Linear time:  $\mathcal{O}(rc)$

Statistics: 29 submissions, 2 accepted, 21 unknown

# I: Interrail Pass

Problem author: Ragnar Groot Koerkamp



**Problem:** Pay trips on  $n$  days  $t_i \in \{0, \dots, T\}$ . The fare for the  $i$ th trip is  $f_i$ . Instead of paying the fare you can use a (multi-ride) *pass*. There are  $k$  types of pass, the  $j$ th has cost  $c_j$  and lasts for a period of  $p_j$  days, during which it covers the first  $d_j$  trips.

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**First step:** Focus on the trip dates  $t_1, \dots, t_n$  (rather than  $\{0, \dots, T\}$ ). Useful to understand the process 'backwards': "if I pay the  $i$ th trip (on day  $t_i$ ) with the  $j$ th pass, then ..."

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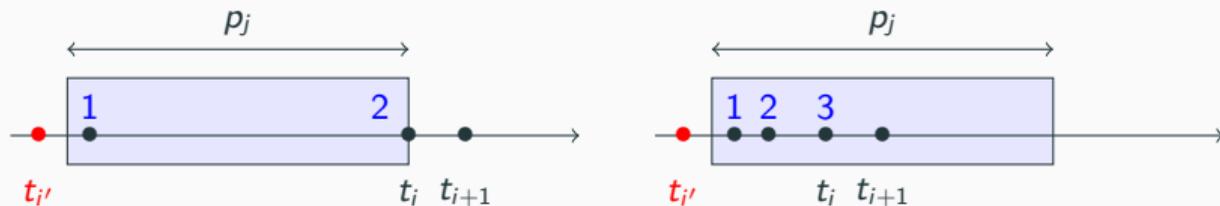
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Can assume that today is the *last* travel day covered by the pass, either because period  $p_j$  ran out before  $t_{i+1}$  or because the pass ran out of days  $d_j$ . E.g., for  $d_j = 3$ :



The index of the first travel day not covered by  $j$ th pass is therefore the largest  $i' \geq 1$  such that  $i' \leq i - d_j$  or  $t_{i'} \leq t_i - p_j$ .

# I: Interrail Pass

Problem author: Ragnar Groot Koerkamp



**Definition:** For  $t \in \{1, \dots, T\}$ , let  $\text{prev}(t)$  be the index of latest trip before day  $t$ , formally

$$\text{prev}(t) = \max\{i: t_i \leq t\}.$$

This can be evaluated in time  $\mathcal{O}(\log n)$  by binary search in  $(t_1, \dots, t_n)$  or in constant time with  $\mathcal{O}(T)$  preprocessing by tabulating  $\text{prev}(t)$  for every  $t$ .

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**Recurrence:** Let  $\text{OPT}(i)$  be the optimum cost for the first  $i$  trips. Then, for  $i > 0$ ,

$$\text{OPT}(i) = \min \begin{cases} f_i + \text{OPT}(i-1), & (\text{pay regular fare}) \\ \min_{1 \leq j \leq k} \{c_j + \text{OPT}(\max(i - d_j, \text{prev}(t_i - p_j)))\} & (j\text{th pass expires today}) \end{cases}$$

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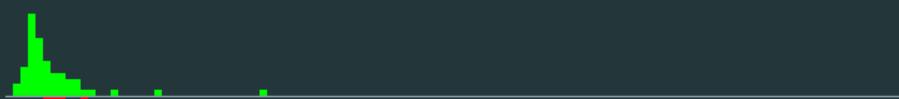
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Statistics: 47 submissions, 20 accepted, 13 unknown

# J: Jumbled Scoreboards

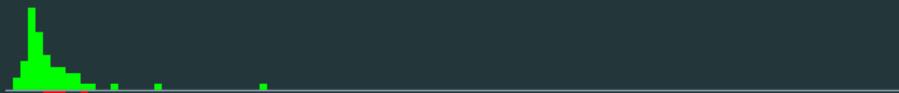
Problem author: Lammert Westerdijk



**Problem:** Given a list of scoreboards, determine whether they are *chronological*: i.e., if these scoreboards can occur in this order in a single match.

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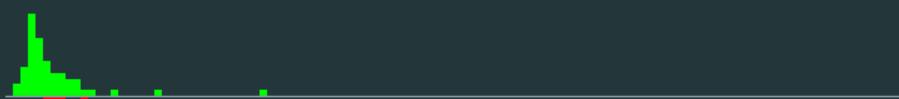


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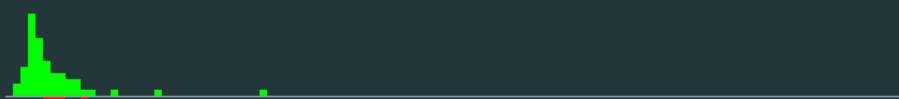
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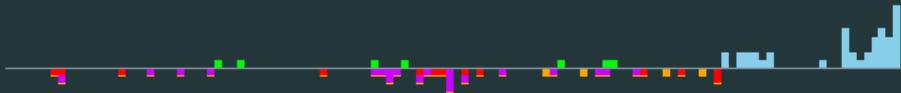
**Solution:** For each list, check whether each pair of consecutive scores is non-decreasing.

**Running time:**  $\mathcal{O}(n)$ .

Statistics: 60 submissions, 56 accepted

# K: Karaoke Compression

Problem author: Jorke de Vlas



**Problem:** Compress a string  $s$  by replacing all occurrences of a chosen substring by a single character, minimizing the total length of the compressed string and the replaced substring.

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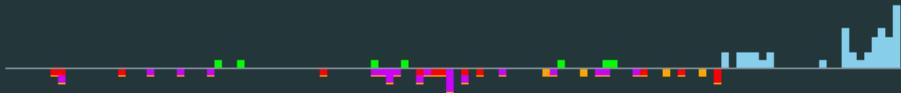


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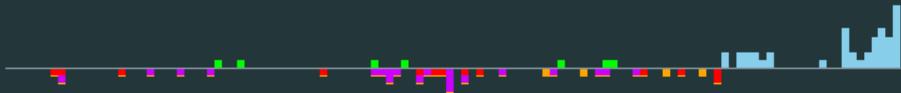
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- Define  $H(c_i c_{i+1} \dots c_j) = c_i b^0 + c_{i+1} b^1 + \dots + c_j b^{j-i} \pmod M$ , where we identify every character with an integer and  $b$  and  $M$  are fixed integers.
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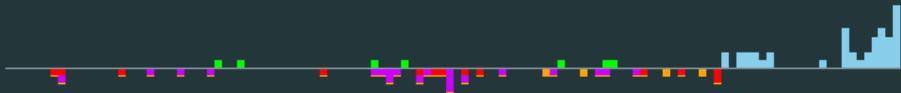
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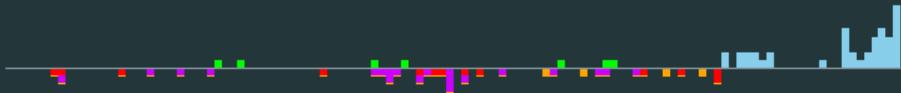
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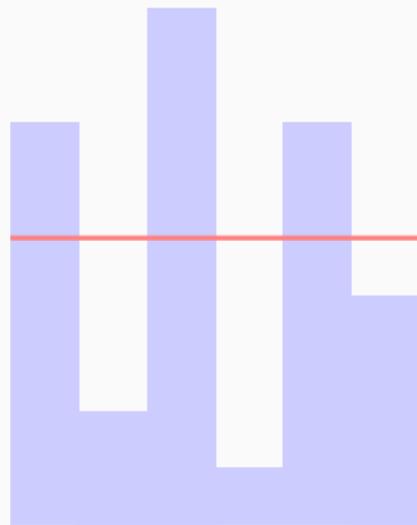
Statistics: 88 submissions, 7 accepted, 43 unknown

# L: Levelling Locks

Problem author: Ragnar Groot Koerkamp



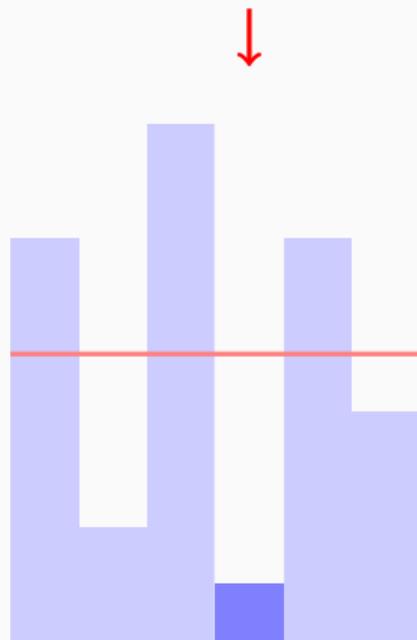
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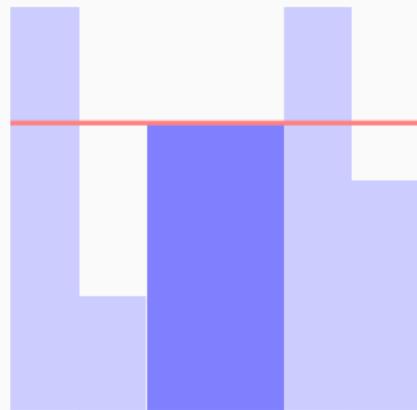


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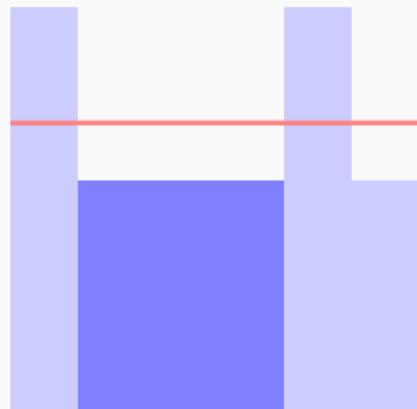


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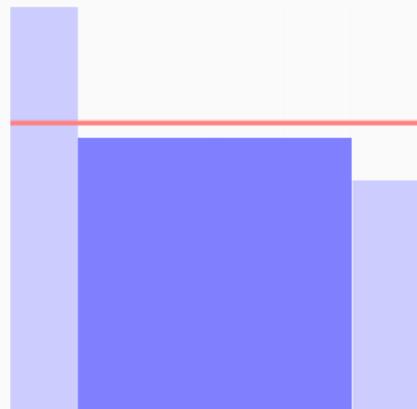


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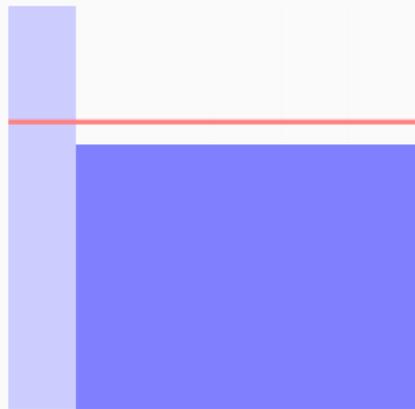


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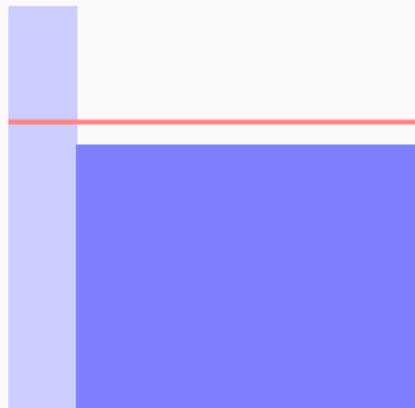


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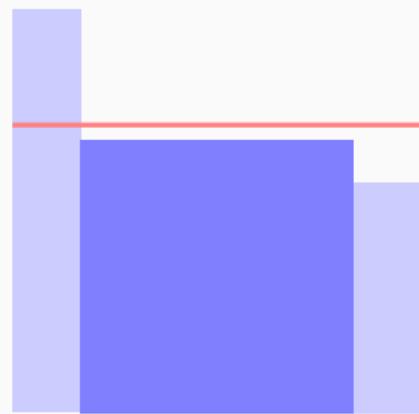


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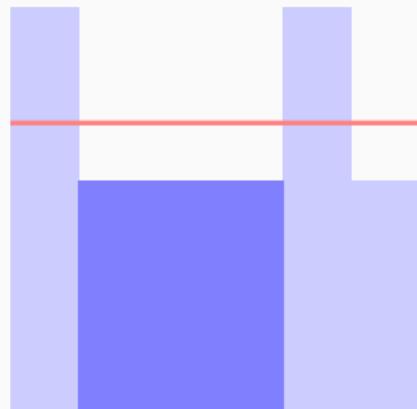
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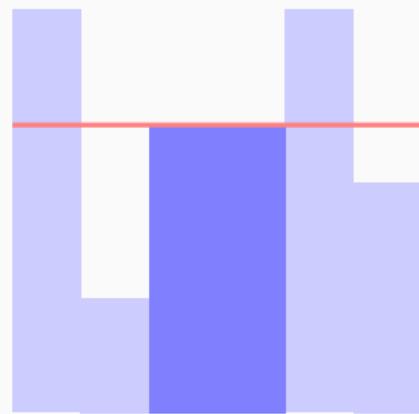


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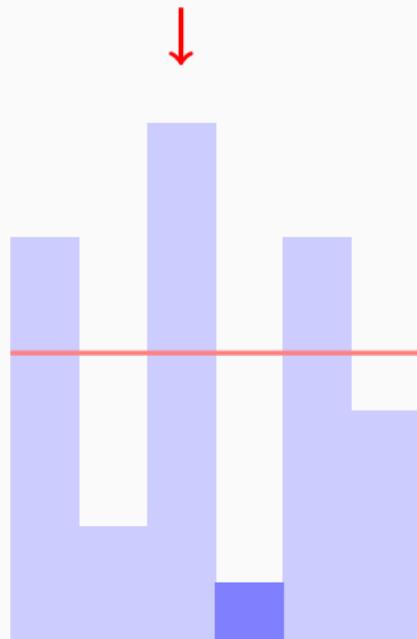
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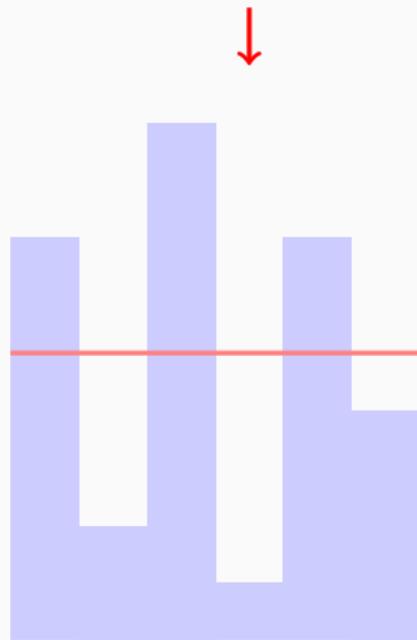
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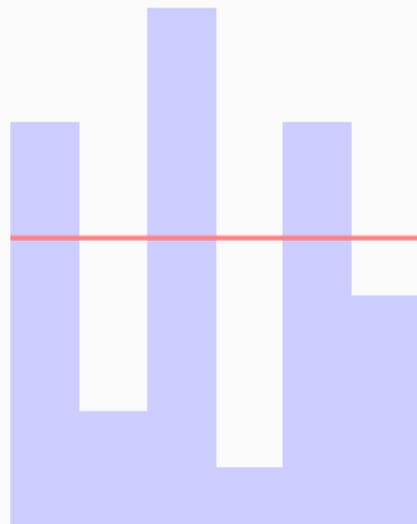
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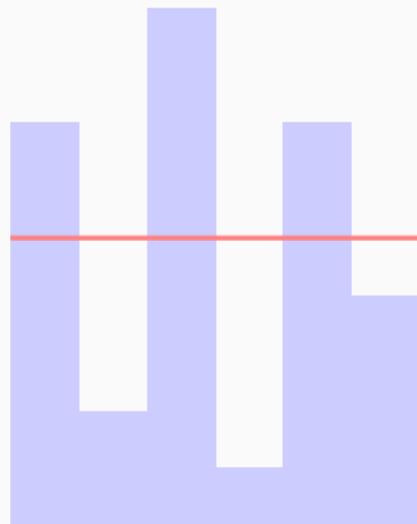


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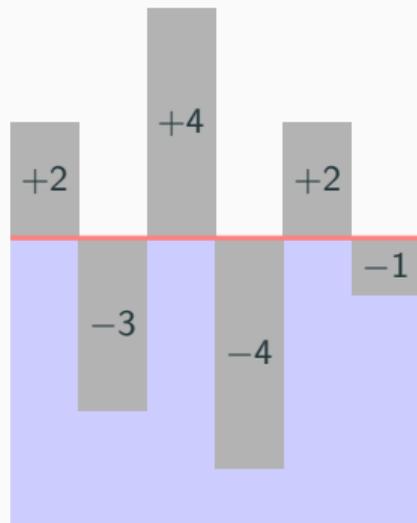
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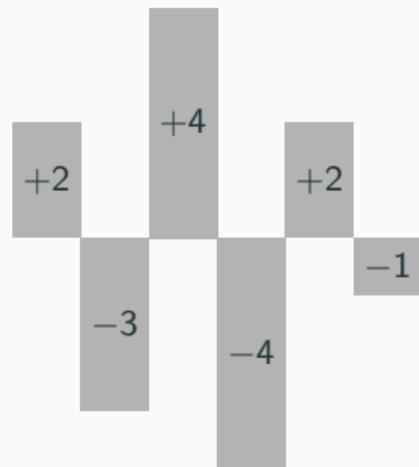


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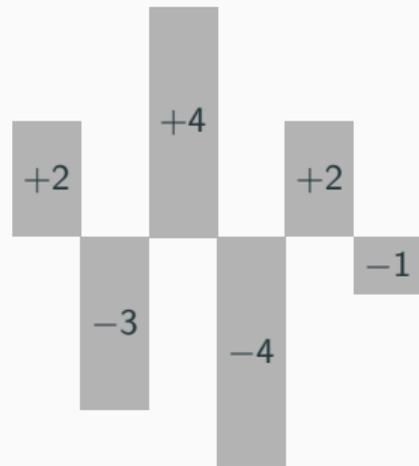
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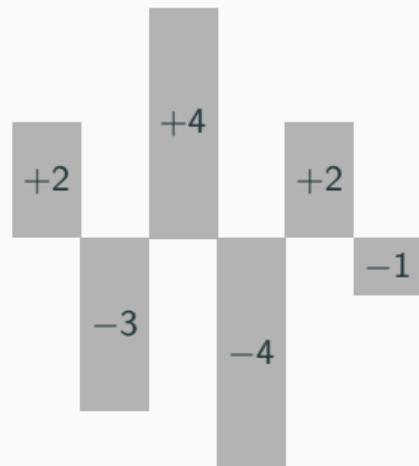


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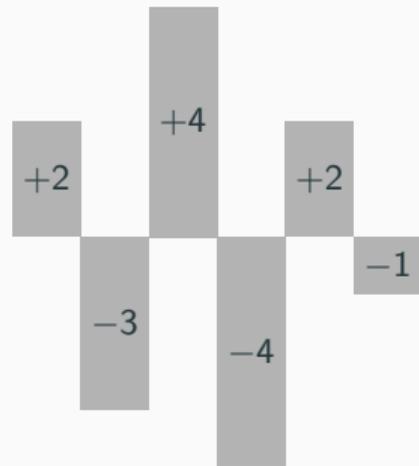


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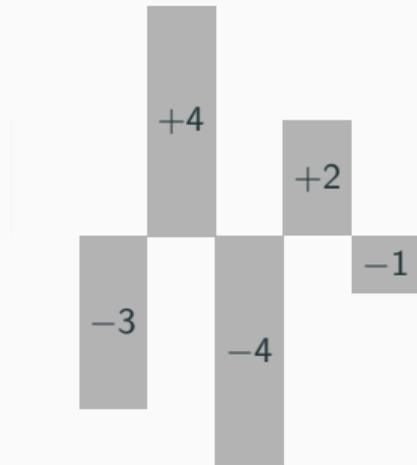


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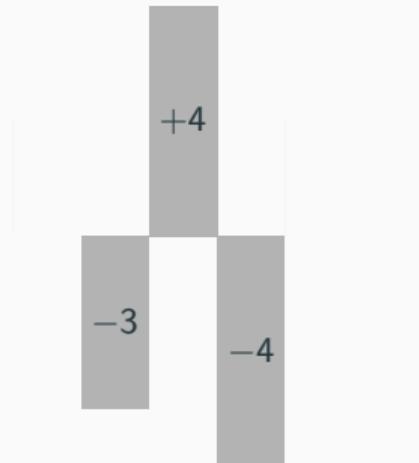


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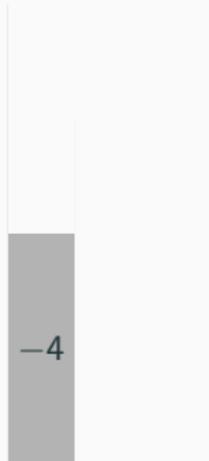


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**Pitfall:** Floating point imprecision. Use integers or resort to fractions.

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+ int sol = (1LL << 55);
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using namespace std;  
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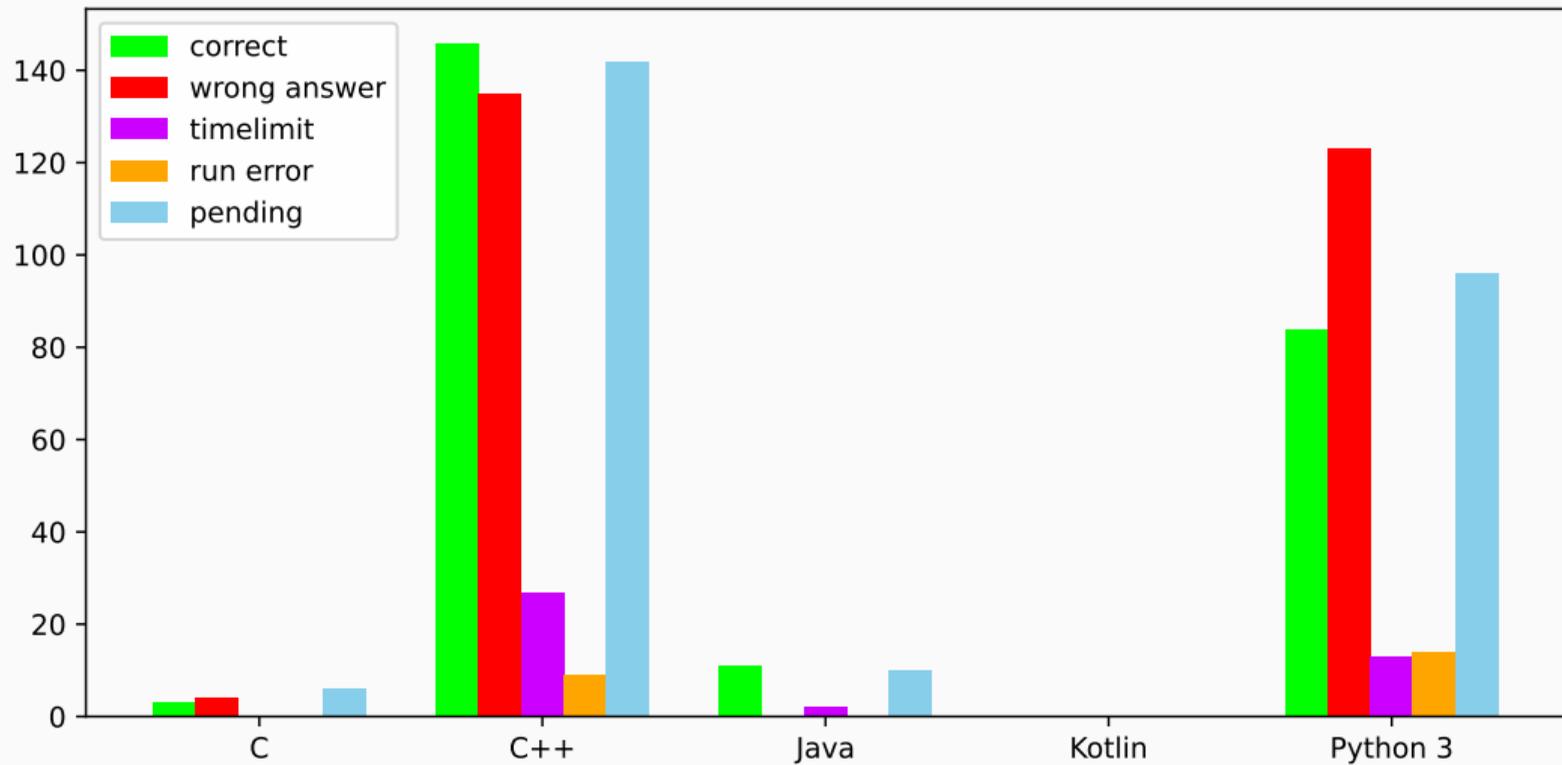
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Statistics: 26 submissions, 5 accepted, 14 unknown

## Language stats



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- The minimum<sup>1</sup> number of lines the jury needed to solve all problems is

$$3 + 10 + 7 + 16 + 3 + 23 + 3 + 16 + 7 + 1 + 2 + 13 + 11 = 115$$

On average, 8.8 lines per problem (7.0 in BAPC 2023, 14.1 in preliminaries 2024)

---

<sup>1</sup>With PEP 8 compliant code golfing

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Yes, this submission for Jumbled Scoreboards is PEP 8 compliant!

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b, a, r, m, f, e = (n := len(s := input() + '$#')), sorted(s[i:] for i in range(len(s))), range, min, next, enumerate
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But this one-liner for Extraterrestrial Exploration is *not*.

```
print('!', 1, (n:=int(input())), [(u:=__import__('functools').reduce(lambda x,y:[x[0],sum(x)//2,x[1]][2*q(sum(x)//2)<v:][:2],x,' '*20,(2,n-1,(q:=lambda x:int(input('? %i\n'%x)))and 0,0*(v:=q(1)+q(n))))[1],u-1][q(u-1)+q(u)>v]]
```

## Thanks to:

### The proofreaders

Arnoud van der Leer

Jaap Eldering

Jeroen Bransen (🔥 Java Hero 📌)

Kevin Verbeek

Michael Vasseur

Mylène Martodihardjo

Pavel Kunyavskiy (🇺🇦 Kotlin Hero 📌)

Wendy Yi

### The jury

Gijs Pennings

Jonas van der Schaaf

Jorke de Vlas

Lammert Westerdijk

Maarten Sijm

Mees de Vries

Mike de Vries

Ragnar Groot Koerkamp

Reinier Schmiermann

Thore Husfeldt

Tobias Roehr

Wietze Koops

Want to join the jury? Submit to the Call for Problems of BAPC 2025 at:

<https://jury.bapc.eu/>