DAPC 2023

Solutions presentation

The BAPC 2023 jury

September 23, 2023

F: Finding Forks

Problem Author: Ragnar Groot Koerkamp

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Note: Can even be solved using 32-bit `int`, because $2 \cdot 10^9 < 2^{31}$.

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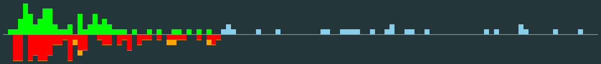
Statistics: 95 submissions, 72 accepted, 13 unknown

B: Better Dice

Problem Author: Mees de Vries



Problem: Determine which of the two custom die is more likely to roll a higher number.

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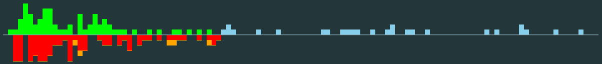

Problem: Determine which of the two custom die is more likely to roll a higher number.

Observation: The dice are fair and only have up to 1000 sides, so we can check all n^2 combinations.

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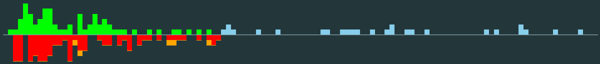

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Solution: For every combination, count whether the first or second die is better.

Compare the total count for both dice to determine which die is more likely to roll a higher number.

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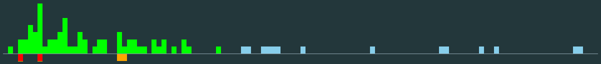
Statistics: 172 submissions, 65 accepted, 26 unknown

J: Just a Joystick

Problem Author: Maarten Sijm



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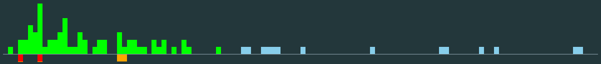

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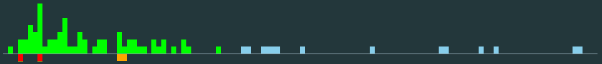

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Solution: Sum, for all pairs of letters, the “distance” on the alphabet wheel:

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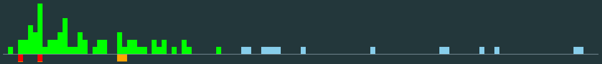

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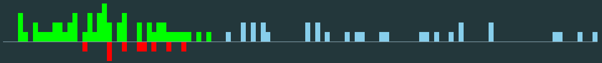
Statistics: 82 submissions, 64 accepted, 14 unknown

K: King's Keep

Problem Author: Ivan Fefer



Problem: Compute the minimal average distance from the most optimal residence keep to the other keeps.

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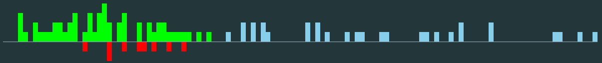

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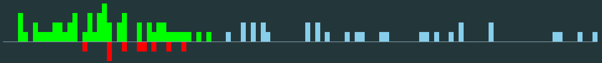

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Observation: There are $k \leq 1000$ keeps, so $\mathcal{O}(k^2)$ is fine.

Solution: For every keep, calculate the average distance to all other keeps, and take the minimum:

$$\min_{1 \leq i \leq k} \left(\frac{\sum_{j \neq i} d(i,j)}{k-1} \right)$$

$d(i,j)$ is Euclidean distance between keeps i and j:

$$d(i,j) = \sqrt{(x_i - x_j)^2 + (y_i - y_j)^2}$$

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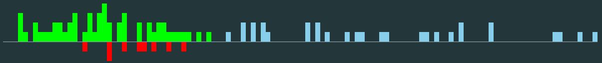

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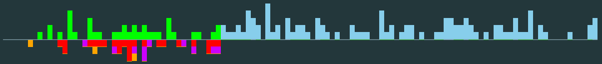
Statistics: 93 submissions, 54 accepted, 30 unknown

I: Idle Terminal

Problem Author: Ragnar Groot Koerkamp



Problem: Calculate the longest time that goes by without seeing a new message on the terminal.

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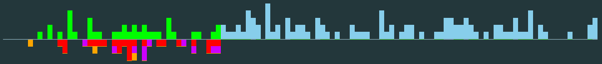

Problem: Calculate the longest time that goes by without seeing a new message on the terminal.

Solution: Simulate the processing of the migration jobs and find the largest gap.

- For each of the n jobs, find the first available CPU core, and update this core's end time.
- Make sure to correctly handle the start and end of the simulation.

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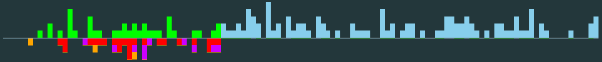

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Pitfall: Finding the first available CPU core in a list ($\mathcal{O}(k)$ time) is too slow,
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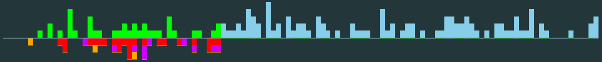

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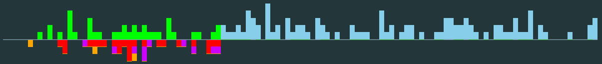

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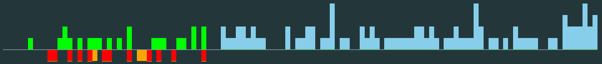
Run time: $\mathcal{O}(n \log k)$

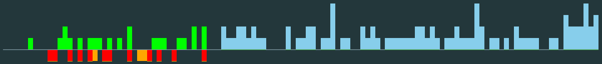
Statistics: 174 submissions, 35 accepted, 101 unknown

A: Anti-Tetris

Problem Author: Maarten Sijm



Problem: Design a *Tetris* grid that perfectly fits the input block.



Problem: Design a *Tetris* grid that perfectly fits the input block.

Observation: The grid can only become perfect if the block has a side with only ‘**#**’.

- (such a side should be at the top)

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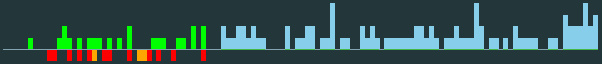

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Rotate the block to have this side point upwards.

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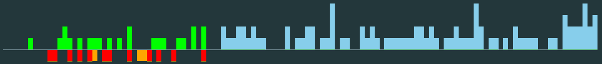

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Rotate the block to have this side point upwards.

Verify that the block has no holes.

- Each column should have only ‘**#**’ at the top, followed by ‘**.**’ at the bottom.

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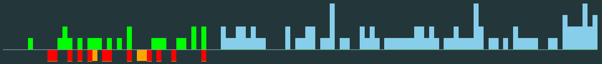

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Invert the block (i.e. swap '**#**' and '**.**') to get a grid that it would fit in.

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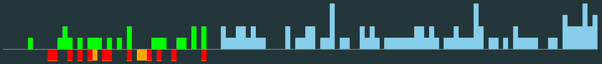

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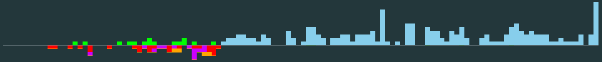
Statistics: 129 submissions, 22 accepted, 91 unknown

C: Cheap Flying

Problem Author: Gregor Behnke



Problem: On the fly, decide whether to use the `airline` or `buy` your own aircraft and fly `yourself`, keeping the cost below twice the optimum.

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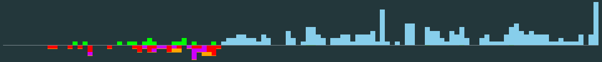

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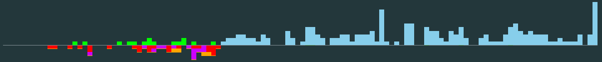
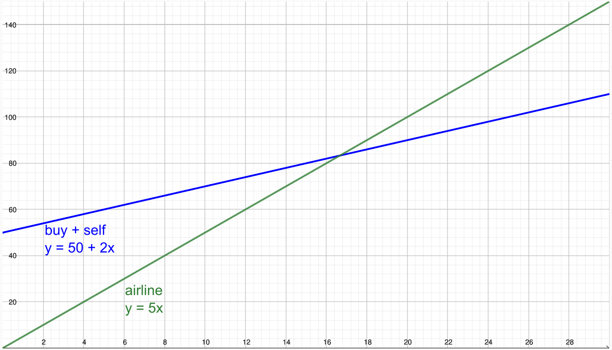


Problem: On the fly, decide whether to use the `airline` or `buy` your own aircraft and fly your`self`, keeping the cost below twice the optimum.

Observation: After `buy`ing your own aircraft, always fly your`self`.

To solve: When to `buy` your aircraft?



Source: Geogebra.org

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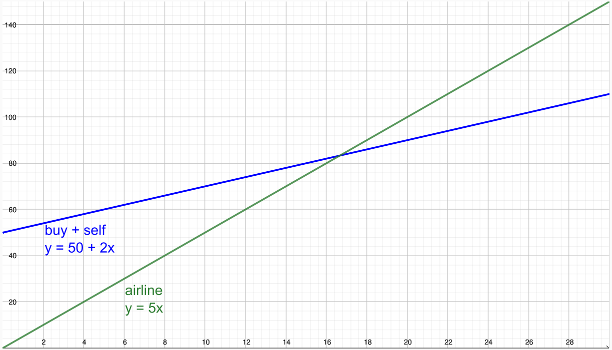
Observation: After `buy`ing your own aircraft, always fly `yourself`.

To solve: When to `buy` your aircraft? $\Rightarrow$ "`buy`" when $b + cx < ax$.



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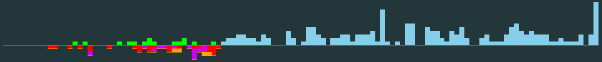

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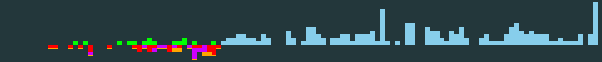

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Edge cases: $0 \leq a, b, c \leq 10^6$, so for example, it is possible that $a > b + c$ (immediately "`buy`") or $a = b = c = 0$ (always "`airline`").

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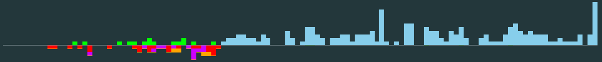

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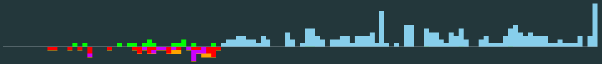

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Statistics: 245 submissions, 15 accepted, 189 unknown

H: Hacky Ordering

Problem Author: Jorke de Vlas

Problem: Find a permutation of the English alphabet such that the strings are sorted.

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Preparation: Instead of running over the entire list of words every time, create a graph, adding edges between the first differing pair of letters of two adjacent words:

```
cplusplus
csharp
python
php
```

c   y

h

$p \longrightarrow s$

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Solution: If the graph contains a cycle, print "`impossible`".
Else, print the reverse order of a post-order traversal of the graph.

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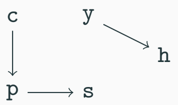
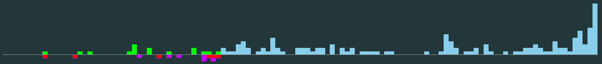


Solution: If the graph contains a cycle, print "`impossible`".
Else, print the reverse order of a post-order traversal of the graph.

Statistics: 117 submissions, 15 accepted, 89 unknown

E: Exceeding Limits

Problem Author: Maarten Sijm



Problem: Find the minimal amount of speeding to arrive on time.

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First check: Perform Dijkstra to check if the destination can be reached on time without speeding.

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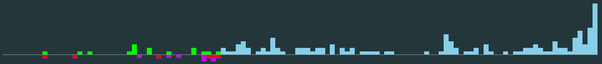

Problem: Find the minimal amount of speeding to arrive on time.

First check: Perform Dijkstra to check if the destination can be reached on time without speeding.

Observation: Driving with different speeds may cause a different route to be faster.

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First check: Perform Dijkstra to check if the destination can be reached on time without speeding.

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Solution: Binary search on the amount of speeding, performing Dijkstra with the new speeds.
If destination can be reached on time, try higher; else, try lower.

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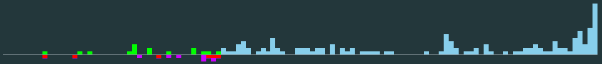

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Run time: $\mathcal{O}((m + n\log m) \cdot \log t)$.

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Note: Floating-point precision is not a problem, because of the low bounds on t and v (10^5).

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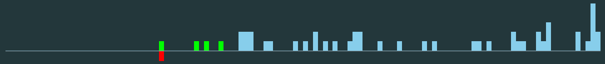
Note: Floating-point precision is not a problem, because of the low bounds on t and v (10^5).

Statistics: 160 submissions, 15 accepted, 133 unknown

G: Gathering Search Results

Problem Author: Pim Spelier



Problem: Given some permutations $\sigma_1, \ldots, \sigma_k$ of $\{1, \ldots, n\}$, determine a permutation such that the total cost is minimized.

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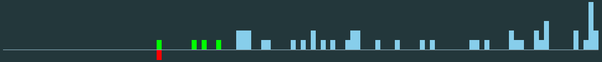

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\end{aligned}$$

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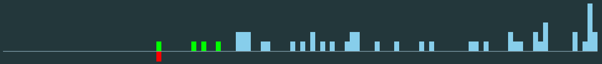

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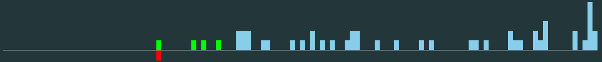

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- So $\sum_{r=1}^{n}(\tau(r) - \mu(r))^2$ needs to be minimized.

Run time: $\mathcal{O}(nk + n\log n)$

Statistics: 51 submissions, 4 accepted, 46 unknown

D: Determining Duos

Problem Author: Pim Spelier

Problem: Given are the scores $x_{t,s}$ of $2n$ students on r topics, where for each topic the scores are a permutation of $\{1, \ldots, 2n\}$.

A pair (team) of students s_1, s_2 has team-score $S(s_1, s_2) := \sum_t \max(x_{t,s_1}, x_{t,s_2})$.

Is it possible to make pairs with total score $\frac{1}{2}rn(3n+1)$.

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Naive: This is general max-weighted matching in a complete graph on $2n$ vertices, where edge s_is_j has weight $S(s_i, s_j)$. (Complicated and too slow.)

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Insight: What is the maximum possible total score per topic? I.e. for a permutation a of $\{1, \dots, 2n\}$, what is the maximum of

$$A = \max(a_1, a_2) + \max(a_3, a_4) + \cdots + \max(a_{2n-1}, a_{2n})?$$

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Swap values such that $a_1 \leq a_2$, $a_3 \leq a_4$, ... Then $A = a_2 + a_4 + \cdots + a_{2n}$, which is maximal when

$$A \leq (n+1) + (n+2) + \cdots + (2n) = \frac{n \cdot ((n+1) + (2n))}{2} = \frac{1}{2}n(3n+1).$$

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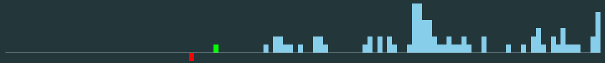

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Thus, $\frac{1}{2}rn(3n+1)$ is exactly the maximal possible score.

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Insight: The maximal pairing is only reached when for each pair of students (s_i, s_j) and each topic t, one of the scores x_{t,s_i} and x_{t,s_j} is *low* ($\leq n$) and the other is *high* ($> n$).

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Solution: First convert the input to binary matrix indicating whether each score is low or high.

$$\begin{matrix} 1 & 4 & 2 & 5 & 2 & 6 \\ 1 & 4 & 5 & 6 & 2 & 3 \\ 1 & 4 & 5 & 6 & 2 & 3 \end{matrix} \quad \longrightarrow \quad \begin{matrix} 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 & 0 \end{matrix}$$

Now we must find a matching between *complementary* columns.

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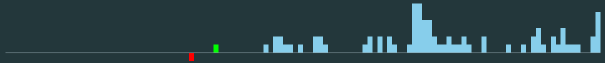

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A matching exists iff each type of column has the same count as its complement.

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Cute trick: Sort the columns, take the complement, and check if this equals the reverse.

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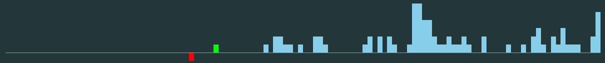

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Run time: $\mathcal{O}(nr\log(n))$.

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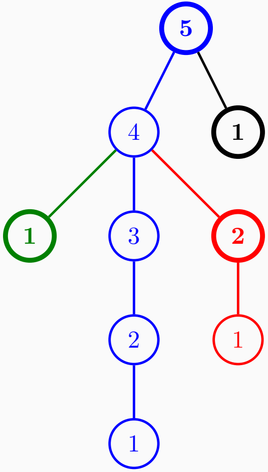
Run time: $\mathcal{O}(nr\log(n))$.

Statistics: 81 submissions, 1 accepted, 79 unknown

Problem: Given a tree of n vertices, remove k of them to minimize the number of remaining leaves.



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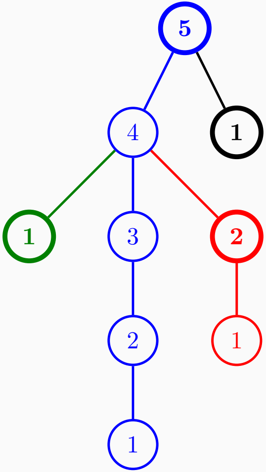
Insight: Removing a leaf only reduces the count if it has siblings.



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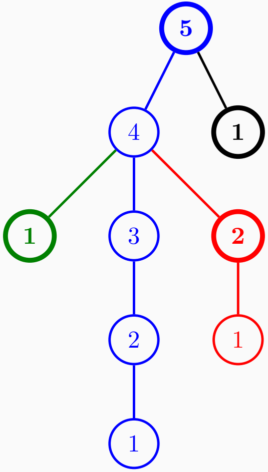
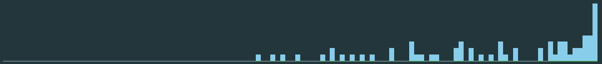
Greedy: Repeatedly remove the shortest *leaf-branch*.





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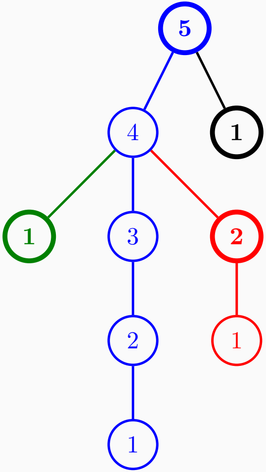
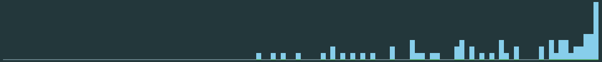
Insight: Below each vertex, the deepest path is always removed last.



L: Losing Leaves

Problem Author: Ragnar Groot Koerkamp



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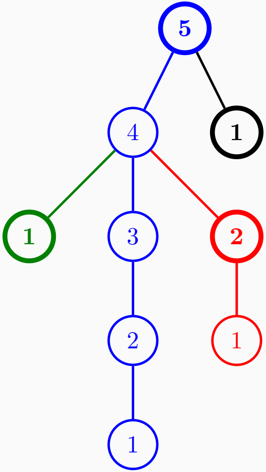
Insight: Below each vertex, the deepest path is always removed last.

Solution: Using DFS or bottom-up DP, find the length of the deepest path below each node.



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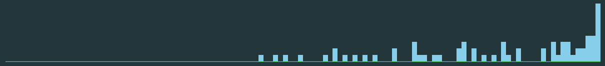

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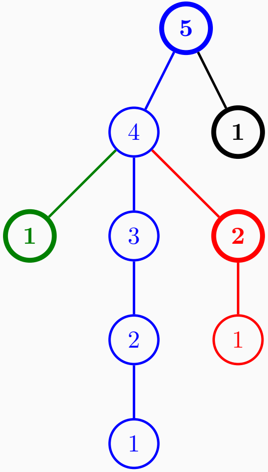
Greedy: Repeatedly remove the shortest *leaf-branch*.

Insight: Below each vertex, the deepest path is always removed last.

Solution: Using DFS or bottom-up DP, find the length of the deepest path below each node.

At each node, increase the length of the deepest child by one, and mark the other children's paths as final (bold).



L: Losing Leaves

Problem Author: Ragnar Groot Koerkamp

Problem: Given a tree of n vertices, remove k of them to minimize the number of remaining leaves.

Insight: Removing a leaf only reduces the count if it has siblings.

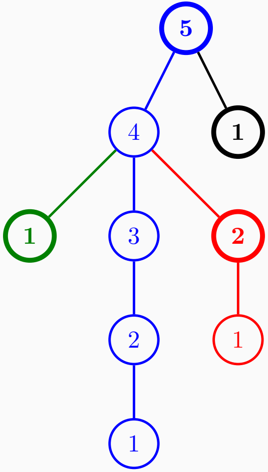
Greedy: Repeatedly remove the shortest *leaf-branch*.

Insight: Below each vertex, the deepest path is always removed last.

Solution: Using DFS or bottom-up DP, find the length of the deepest path below each node.

At each node, increase the length of the deepest child by one, and mark the other children's paths as final (bold).

Sort the final lengths ($[1, 1, 2, 5]$), and count how many of them sum to at most k.



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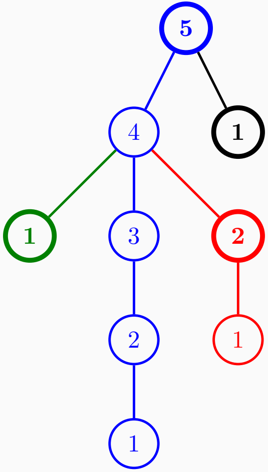
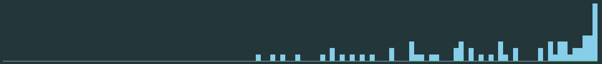
Run time: $\mathcal{O}(n)$.





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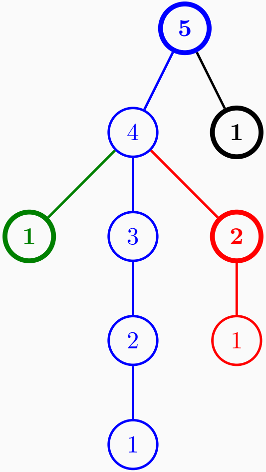
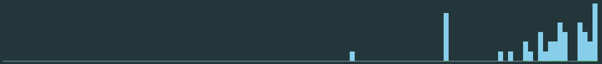


Statistics: 69 submissions, 0 accepted, 69 unknown

M: Monorail

Problem Author: Ragnar Groot Koerkamp



Problem: Given $n \leq 500$ trains that arrive at the north/south end of a one-lane tunnel, determine the minimal total waiting time.

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Insight: A train enters the tunnel either:

- *On time*: as soon as it arrives, or
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Insight: After a train exits the tunnel, there are four possibilities for the next train:

1. Same direction and departs on time.
2. Opposite direction and enters at a later time (always on time).
3. Same direction and departs late, at the same time as current train.
4. Opposite direction and enters directly after (on time or late).

M: Monorail

Problem Author: Ragnar Groot Koerkamp

Solution: *Forward* DP: $DP[d][i][j]$ is the minimal total waiting time for the first i trains going north and j trains going south where the last train is in direction d and leaves on time.

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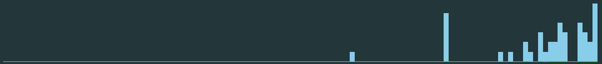

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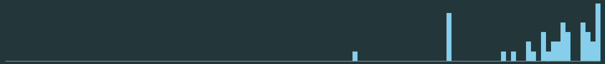

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Expand: Given state (N, i, j, T, W): i trains going north done; j trains going south done; last train went north and entered at time T; total waiting time W. Next possible states:

E1. $DP[N][i+1][j] \leq W$, when the next northbound train is on time $(N_{i+1} \geq T)$;

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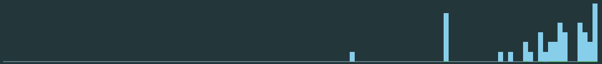

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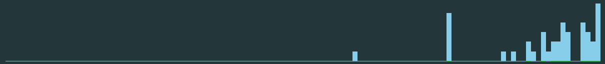

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(See jury submissions for details.)

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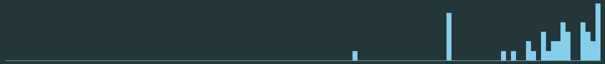

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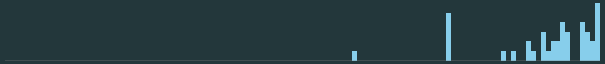

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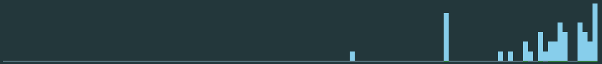

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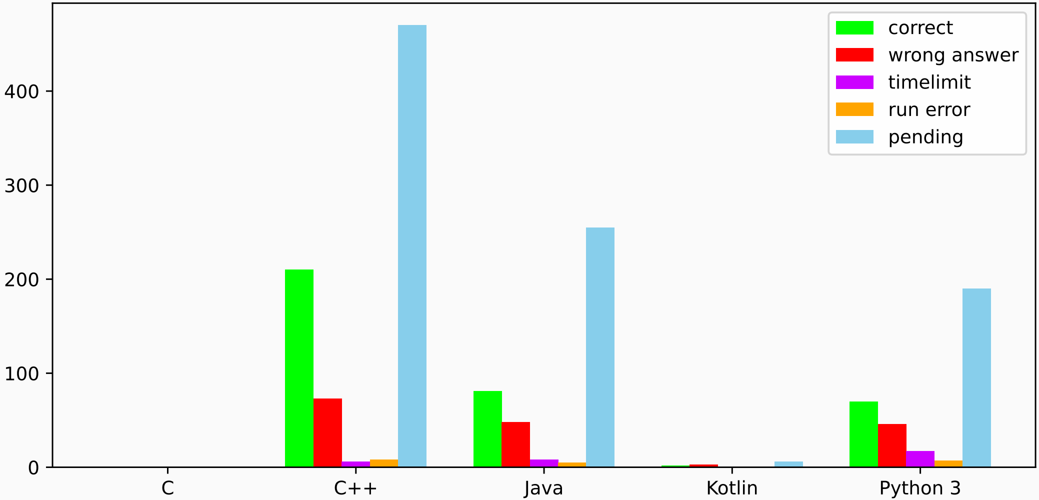
Statistics: 41 submissions, 0 accepted, 41 unknown

Language stats



Random facts

Jury work

- 492 commits (last year: 285)

[1] With limited codegolfing

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- The minimum[1] number of lines the jury needed to solve all problems is

$$5 + 14 + 15 + 4 + 21 + 2 + 10 + 30 + 9 + 2 + 3 + 18 + 48 = 181$$

  On average 13.9 lines per problem, up from 6.6 in last year's preliminaries

[1] With limited codegolfing

ETV also did their best!

Thanks to:

The proofreaders

Angel Karchev
Boas Kluiving
Jaap Eldering
Kevin Verbeek
Mark van Helvoort (Java Hero)
Michael Vasseur
Michael Zündorf
Nicky Gerritsen (Java Hero)
Paul Wild
Pavel Kuvnyavskiy (Kotlin Hero)
Thomas Verwoerd (Kotlin Hero)

The jury

Gregor Behnke
Ivan Fefer
Jorke de Vlas
Ludo Pulles
Maarten Sijm
Mees de Vries
Mike de Vries
Ragnar Groot Koerkamp
Reinier Schmiermann
Wessel van Woerden