BAPC 2023

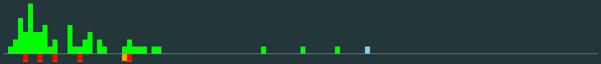
Solutions presentation

The BAPC 2023 jury

October 28, 2023

D: Democratic Naming

Problem Author: Ivan Fever



Problem: Given the list of city names, determine the new county's name based on the existing city names.

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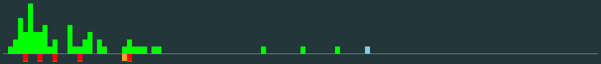

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Observation: Every letter can be handled individually.

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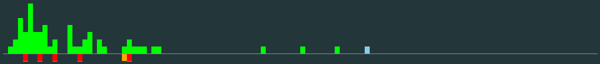

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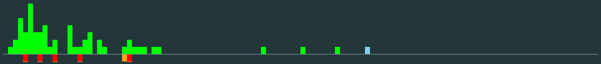

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Statistics: 63 submissions, 56 accepted, 1 unknown

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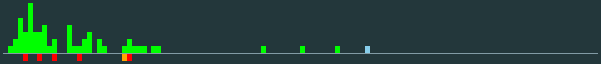

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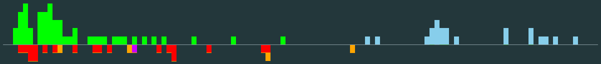
(spoiler: they solved it! 🎈)

A: APT Upgrade

Problem Author: Ragnar Groot Koerkamp



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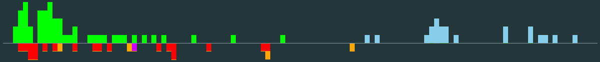

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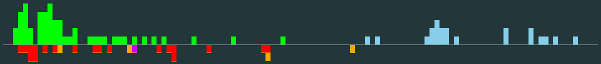

Problem: Calculate the maximum overall completion percentage of downloading n packages, with m packages having finished downloading and k packages underway.

Observation 1: The largest packages need to have finished downloading.

Observation 2: The packages underway need to be the next largest, and at $99.\bar{9}\%$.

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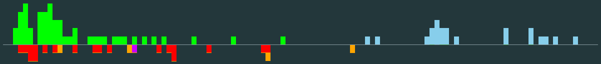

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Solution: Sort the list, sum the largest $m + k$ packages, divide by the total sum, multiply by 100:

$$\frac{\sum_{i=1}^{m+k} s_i}{\sum_{i=1}^{n} s_i} \cdot 100 \qquad \text{(assuming } s_i \text{ are sorted from large to small)}$$

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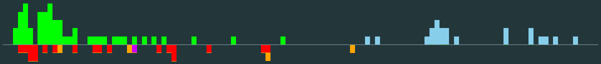

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Statistics: 95 submissions, 50 accepted, 21 unknown

B: Battle Bots

Problem Author: Mees de Vries

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Naive solution: Just try all possible combinations: S, C, SS, SC, CS, CC, SSS, SSC, ..., until you find one that works.

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Naive solution: Just try all possible combinations: S, C, SS, SC, CS, CC, SSS, SSC, $\ldots$, until you find one that works.

If m is the answer, this runs in $\mathcal{O}(m2^m)$. Since $m \approx \log_2(n)$, this is $\mathcal{O}(n\log(n))$. Too slow!

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Observation: An optimal strategy is to use a series of S attacks followed by a single C.

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Solution: Use claw attacks until the remaining size is < 1, then a single claw. Run time: $\mathcal{O}(\log(n))$

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Solution: You can also compute the answer directly as $\lceil \log_2(n) \rceil + 1$, but only if you either

1. Use `long double` in C++, which has 18 digits of precision
2. Calculate the bit length (in Python: `(x - 1).bit_length() == ceil(log2(x))`)

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Float note: 64-bit floating-point numbers (`double`) have too low precision (only 15 digits).

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Statistics: 127 submissions, 50 accepted, 12 unknown

F: Funicular Frenzy

Problem Author: Ragnar Groot Koerkamp

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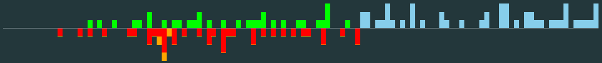
Statistics: 111 submissions, 45 accepted, 18 unknown

G: Geometry Game

Problem Author: Jorke de Vlas



Problem: Determine the *most restrictive* type of quadrilateral from four points.

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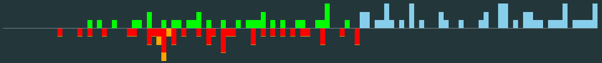

Problem: Determine the *most restrictive* type of quadrilateral from four points.

Possible solution: There are multiple ways of determining the shapes, this is one of them:

- If all four sides have equal length, output “`square`” if the two diagonals have equal length, else “`rhombus`”.
- If two pairs of opposite sides each have equal length, output “`rectangle`” if the two diagonals have equal length, else “`parallelogram`”.
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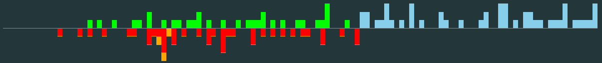

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Parallel test: Check if out-product of two vectors equals zero:

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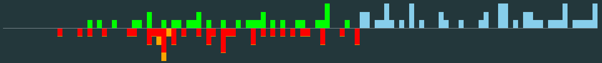

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Float note: Calculating the length of an edge ($\sqrt{x^2 + y^2}$) requires 18 digits (59 bits) of precision. `double` only has 53!
I.e. 64-bit integers (without $\sqrt{}$) or C++ `long double` with an epsilon of 10^{-19} works.

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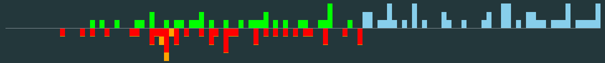

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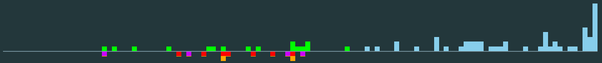
Statistics: 122 submissions, 32 accepted, 48 unknown

Problem: Which working directory should you use to specify n file paths (with ../), with the minimal number of relative path components?



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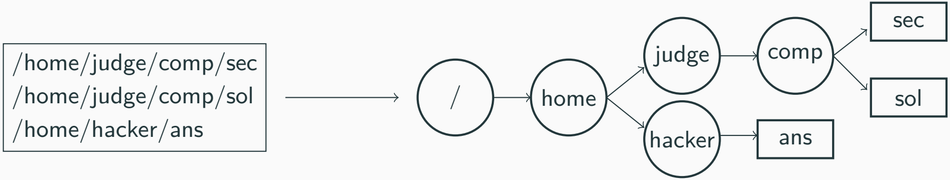
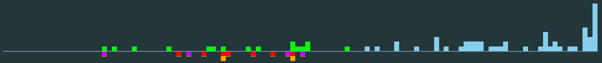


C: Compressing Commands

Problem Author: Ragnar Groot Koerkamp



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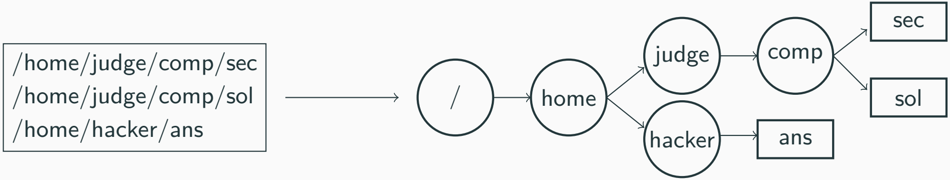

Compute #path components for all nodes in linear time.

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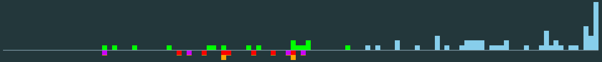

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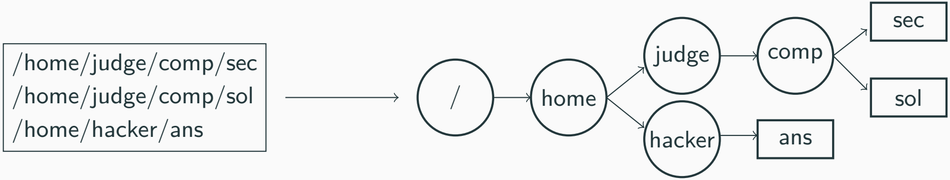

Compute #path components for all nodes in linear time.

1. $cost("/") = \#\text{total\_path\_components}$.
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3. Output $\min_u cost(u)$.

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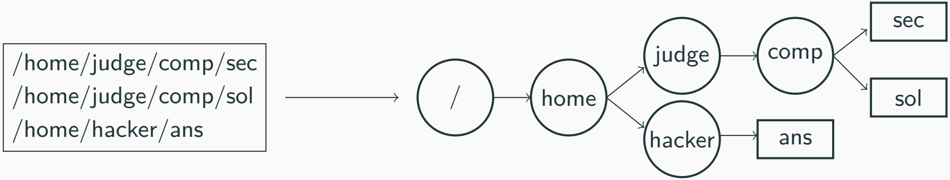

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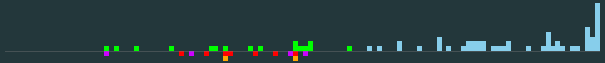

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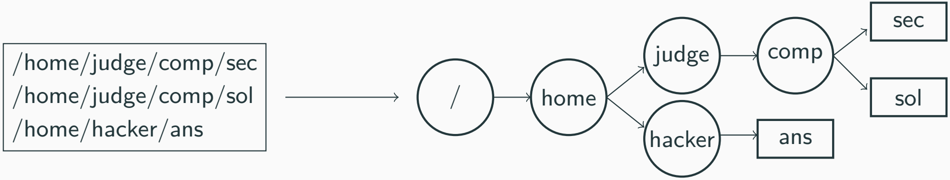

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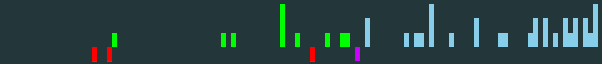
Statistics: 82 submissions, 16 accepted, 53 unknown

Problem: Given an exam schedule, determine how many exams you can pass with optimal scheduling.

E: Exam Study Planning

Problem Author: Jorke de Vlas and Reinier Schmiermann



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Greedy approach: Study for the first exam you can pass. Doesn’t work: maybe you can study for more shorter exams. (Sample 2!)

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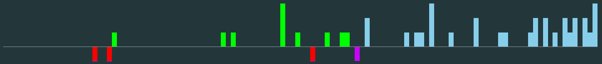

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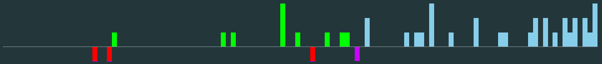

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Brute force: Try all pass/fail combinations: runs in $\mathcal{O}(2^n)$, way too slow.

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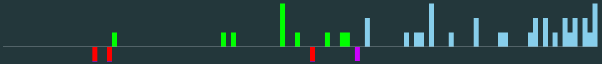

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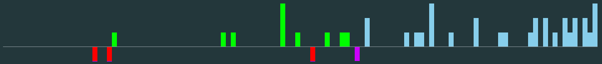
Brute force: Try all pass/fail combinations: runs in $\mathcal{O}(2^n)$, way too slow.

Observation: If at time e_i, end time of exam i, you have passed j exams, and have x minutes of study time unused, it doesn't matter which j exams you passed!

E: Exam Study Planning

Problem Author: Jorke de Vlas and Reinier Schmiermann



Problem: Given an exam schedule, determine how many exams you can pass with optimal scheduling.

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Use dynamic programming:

$$\mathrm{DP}(i,j) = \begin{cases} x, & \text{max extra study time at } e_i \text{ with } j \text{ exams passed,} \\ -\infty & \text{if it's impossible to pass } j \text{ exams at } e_i. \end{cases}$$

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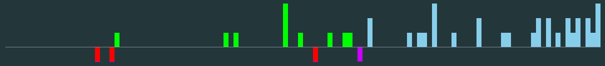

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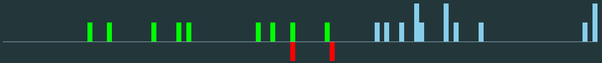
Statistics: 44 submissions, 10 accepted, 30 unknown

L: Locking Doors

Problem Author: Jorke de Vlas and Mike de Vries



Problem: Given the layout of a building, with doors that lock from only one side, how many exits on the outside do we need to close all doors?

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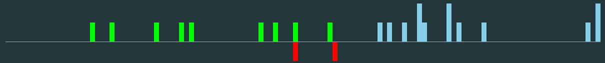

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Observation: If a room *a* has an exit, then which doors can we close using that exit?

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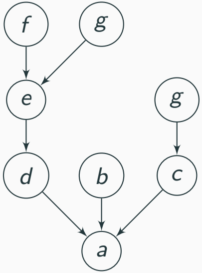
Write $b \to a$ to mean there is a door you can close from side a. Then consider:



If there is an exit at a, you can close all these doors: just start at any leaf, close that door, and repeat.

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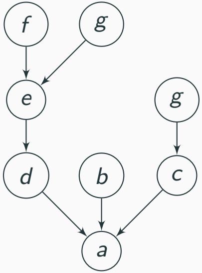

If there is an exit at *a*, you can close all these doors: just start at any leaf, close that door, and repeat.

Maybe you can close *more* doors, but definitely these ones.

Strongly connected: For a, b nodes, if you can walk from a to b via arrows, and also b to a, we call a and b *strongly connected*.

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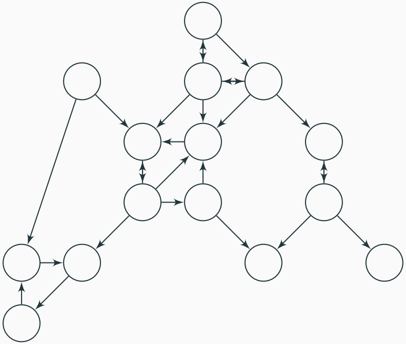

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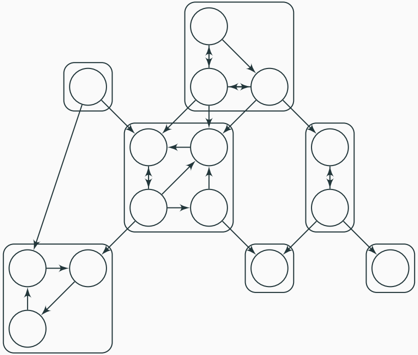

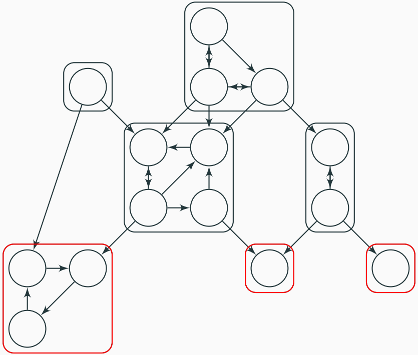

Necessary How many exits does this graph need? We need *at least one* in the (red) components without outgoing edges. Otherwise you can never leave it once you close the last incoming door.

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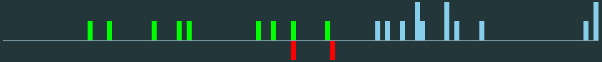

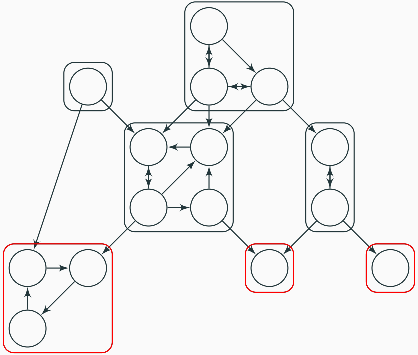

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Sufficient That is also *enough exits*: from any node you can follow the arrows to one of those components, which we saw is enough to close all doors.

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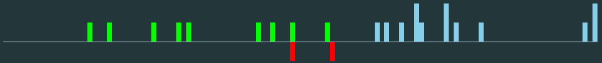

Solution Find the strongly connected components, e.g. with Tarjan's algorithm. Output the number of SCCs without outgoing edges.

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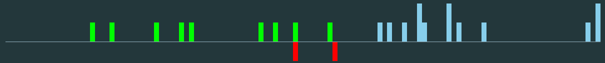

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Since we only have to count root-SCCs, simpler algorithms are also possible.

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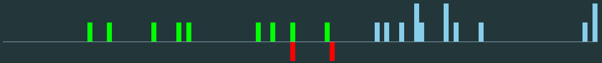

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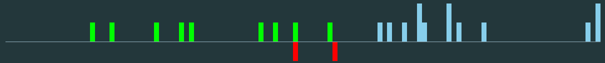

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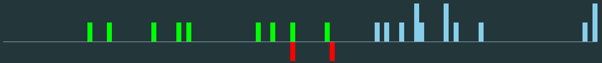

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Statistics: 24 submissions, 9 accepted, 13 unknown

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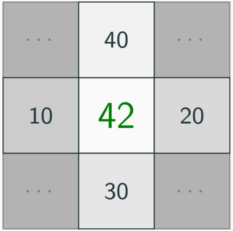

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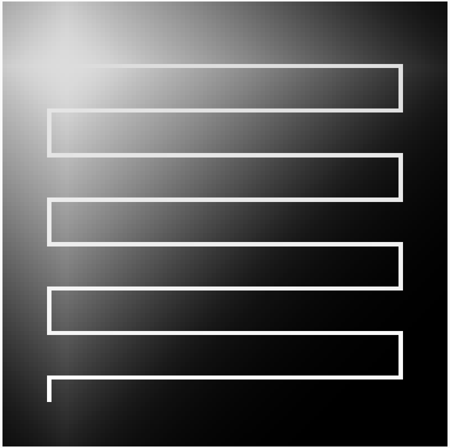

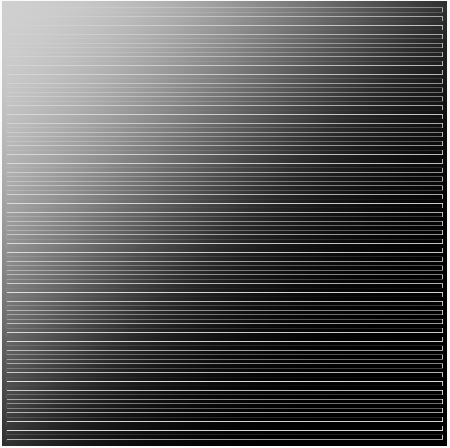

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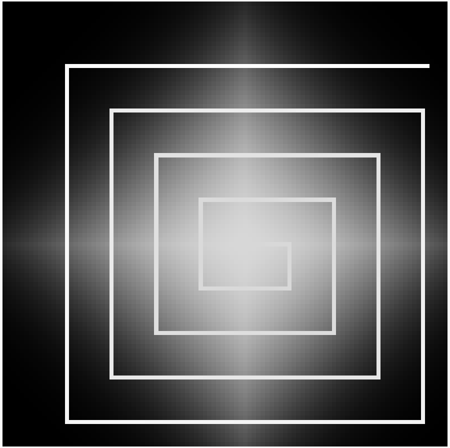

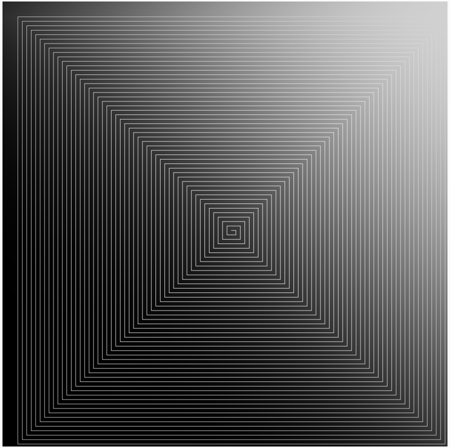

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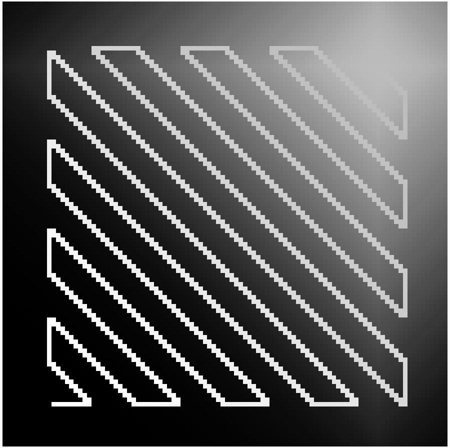

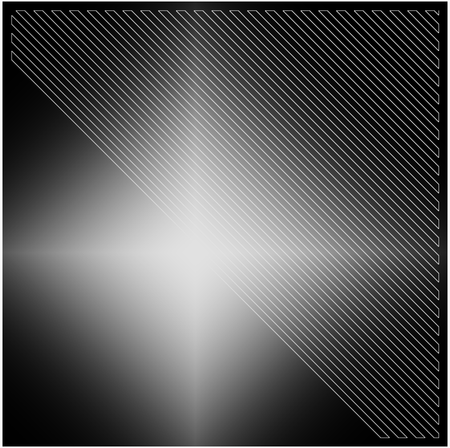

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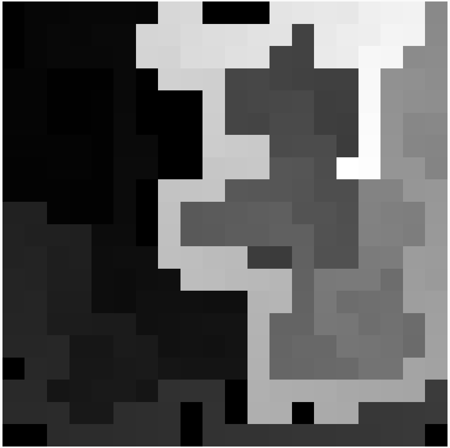

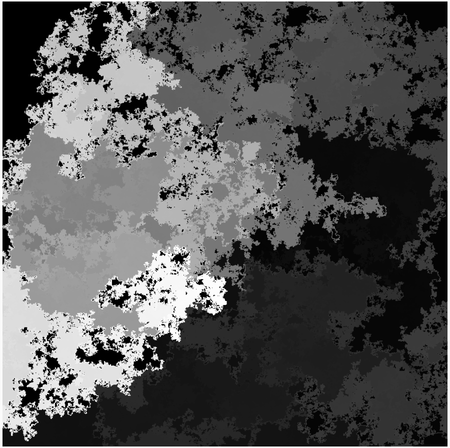

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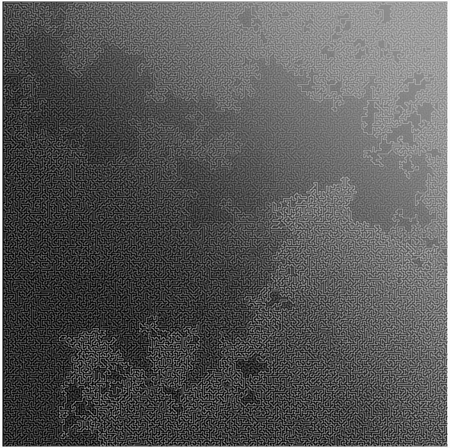

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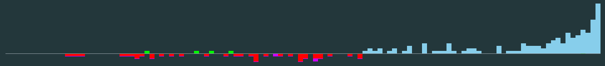

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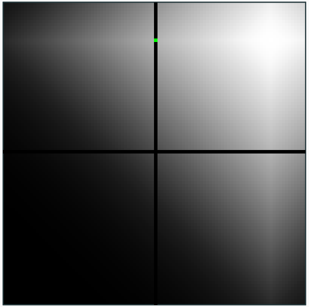

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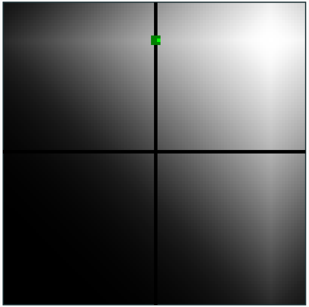

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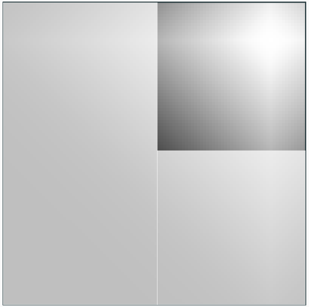

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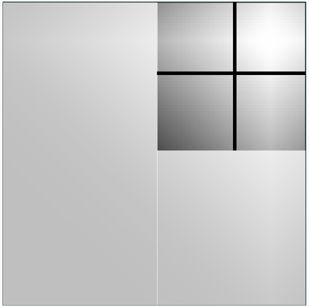

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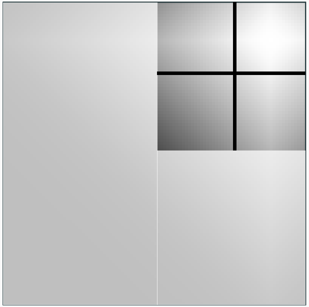

Number of queries: $\approx 2n + n + \frac{1}{2}n + \cdots \approx 4n$

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- If you find a point in the top-$2n$, you need at most $6n$ hill climbing queries to find the absolute maximum (think of the worst case: sparse zigzag/spiral).

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- For every query, the probability of hitting a point that is in the top-$2n$ is $\frac{2n}{n^2} = \frac{2}{n}$.

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K: King of the Hill

Problem Author: Maarten Sijm

Problem: Find the highest value in an n^2 grid in $10n + 100$ queries ($n \leq 10\,000$).

Given: There is only one local maximum.

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Statistics: 186 submissions, 4 accepted, 141 unknown

H: Hidden Art

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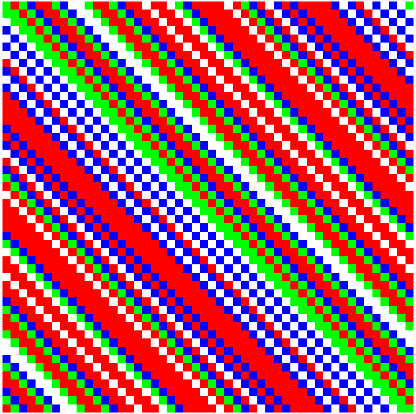

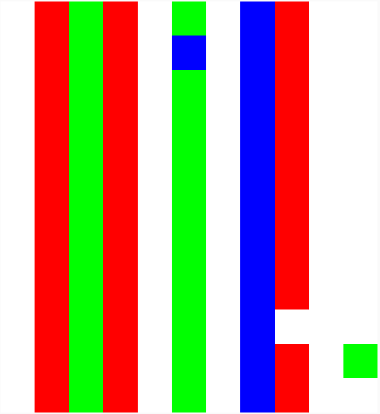

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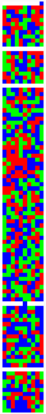

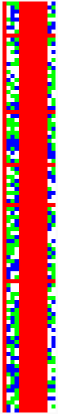

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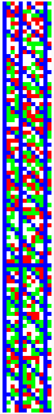

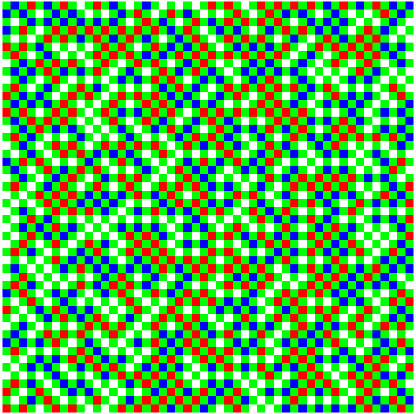

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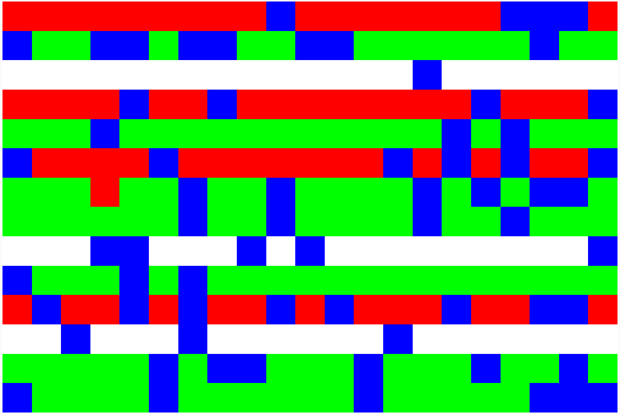

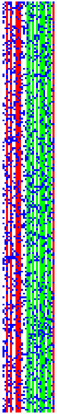

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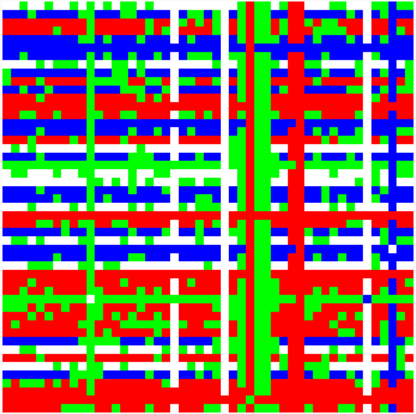

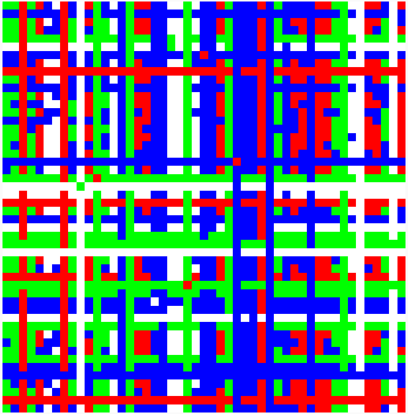

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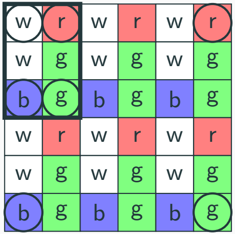

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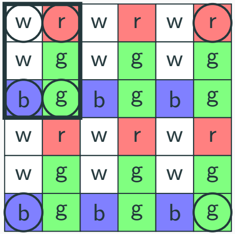

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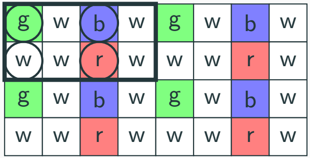

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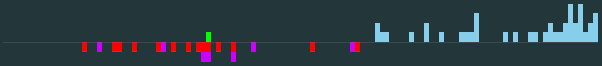

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In the example above: it doesn't work, because $x - y = 2 - 1 = 1$ while $\gcd(w, h) = \gcd(4, 2) = 2$.

Naive solution: For every rectangle in the grid, check if its corners have all four colors, and if the difference between height and width is divisible by $g = \gcd(w, h)$.

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Observation: There are not that many combinations of colors possible.

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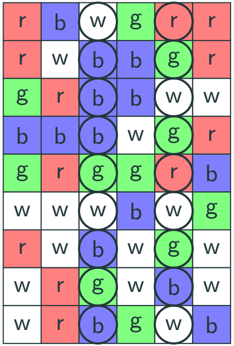

Row 0 (mod 3): { w r b g b g }

Row 1 (mod 3): { b g g r g b }

Row 2 (mod 3): { b w w w b w }

Solution: Fix two columns. Then check all colour combinations in those two columns, and store them by their row $(\text{mod } g)$.

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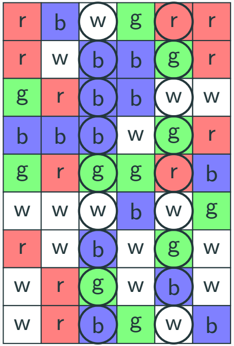

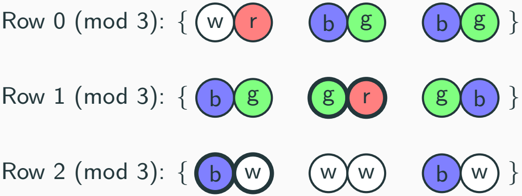

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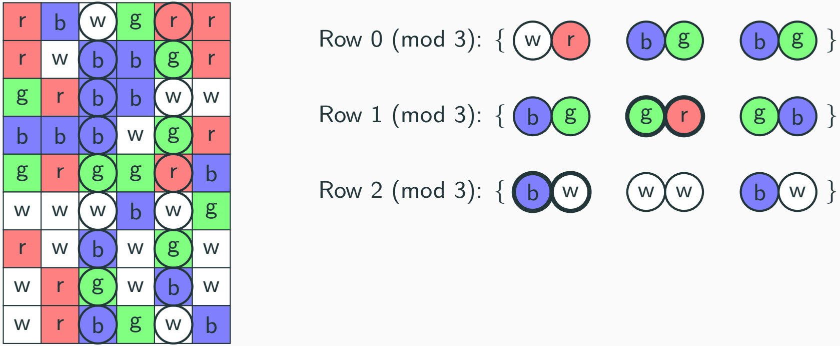

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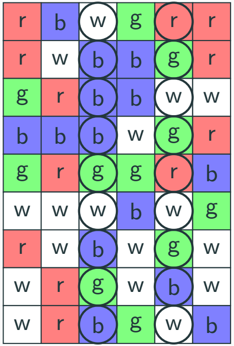

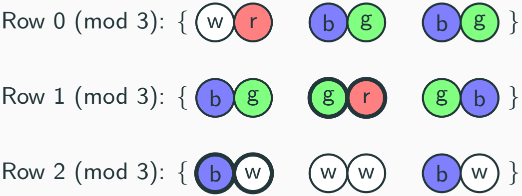

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Statistics: 63 submissions, 1 accepted, 41 unknown

J: Jungle Job

Problem Author: Jorke de Vlas and Mike de Vries

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If we can compute F, we can get the answer to the problem by calculating $\sum_{i \in V} F(i, c)$.

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Solution: Use dynamic programming

Base case: If v is a leaf:

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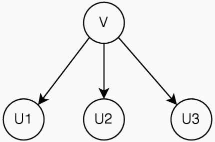

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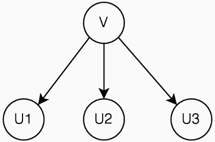

To calculate $F(v, c)$ we need to consider every way to distribute $c - 1$ remaining nodes among three child subtrees of v:

$$F(v, c) = \sum_{c_1=0}^{c-1} \sum_{c_2=0}^{c-1-c_1} F(u_1, c_1) F(u_2, c_2) F(u_3, c - c_1 - c_2)$$

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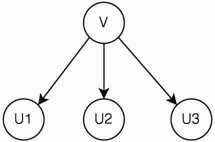

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Problem: For a node with many children m, this will hit the time limit:

$$F(v, c) = \sum_{c_1=0}^{c-1} \sum_{c_2=0}^{c-1-c_1} \dots \sum_{c_{n-1}=0}^{c-1-\dots} \prod_{i=1}^{m} F(u_i, c_i)$$

Fix: Introduce $F'(v, i, c)$ - the number of connected subtrees, that have node v as the root, have exactly c nodes and only include first i children of node v.

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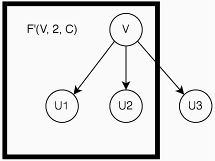
Base cases for node v that has m children:

$F(v, c) = F'(v, m, c)$,

$F'(v, 1, c) = F(u_1, c - 1)$,

DP Let's calculate $F'(v, 2, c)$ for this graph:



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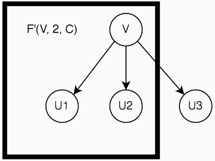
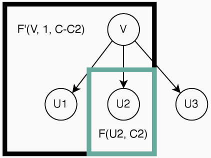


For that we just need to decide how many nodes will be in the subtree of the second child and then we can recurse: $F'(v, 2, c) = \sum_{c2=0}^{c-1} F'(v, 1, c - c_2)F(u_2, c_2)$



Runtime: Computing $F'(v, i, c)$ for all c takes $O(|u_i| \cdot \sum_j |u_j|)$ time, where $|u_i|$ denotes the size of the subtree at u_i.

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Observation: $\sum_i \sum_j |u_i| \cdot |u_j|$ is the number of pairs of nodes with lowest common ancestor v.

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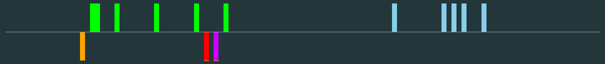

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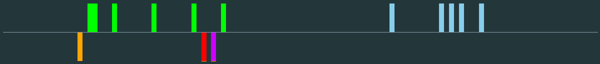

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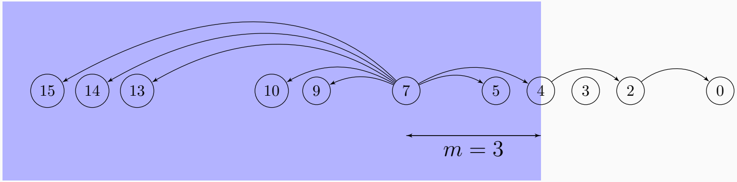
Statistics: 14 submissions, 6 accepted, 5 unknown

Problem: Given are $n \leq 10^5$ countries with ascending infection rates r_i, and quarantine times t_i.



- *Hop*: if $r_j \geq r_i - m$, go without quarantine (1 day).
- *Jump*: go with quarantine ($1 + t_j$ days).

Answer 10^5 queries: What is the fastest route from x to y.

Solution If $r_x < r_y$: We can hop directly, so print 1.

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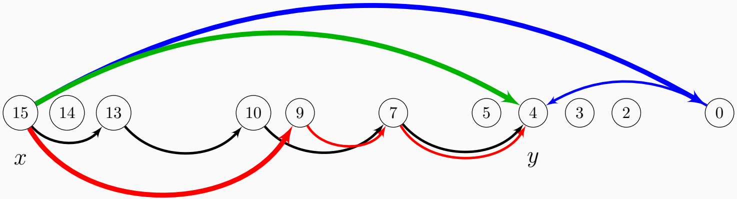
If $r_x > r_y$, four options:

1. *Hop* to the right up to m at a time.
2. *Jump* directly to y.
3. *Jump* right of y, then hop left once.
4. *Jump* left of y, then hop right some times.



Case 1: Hop to the right up to m at a time.

Define $H_k(i)$ as the rightmost country reachable within 2^k hops.

I: International Irregularities

Problem Author: Ragnar Groot Koerkamp

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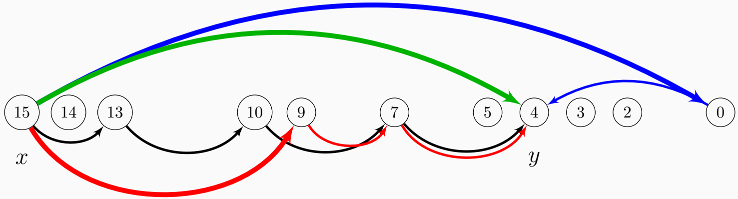
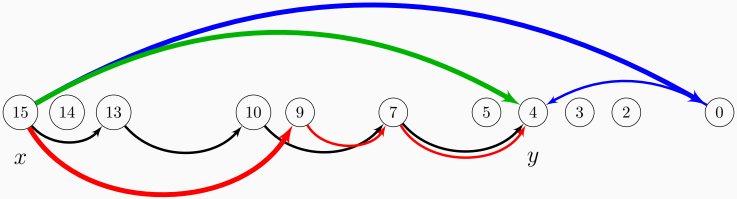
Add one for the hop.

Case 4: Hop to the left of y, then hop right some times.



Iterate through the countries from left to right, keeping track of the best country to jump to first.

Case 4: Hop to the left of y, then hop right some times.



Iterate through the countries from left to right, keeping track of the best country to jump to first.

For each country, either:

- jump to the stored best and hop from there, or
- jump directly and update the stored best.

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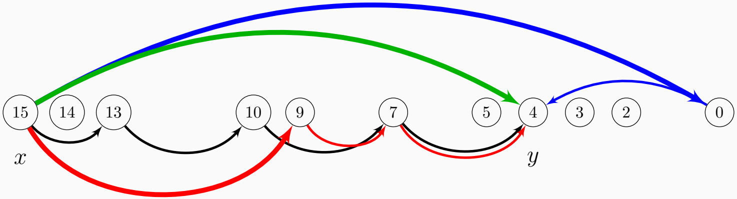

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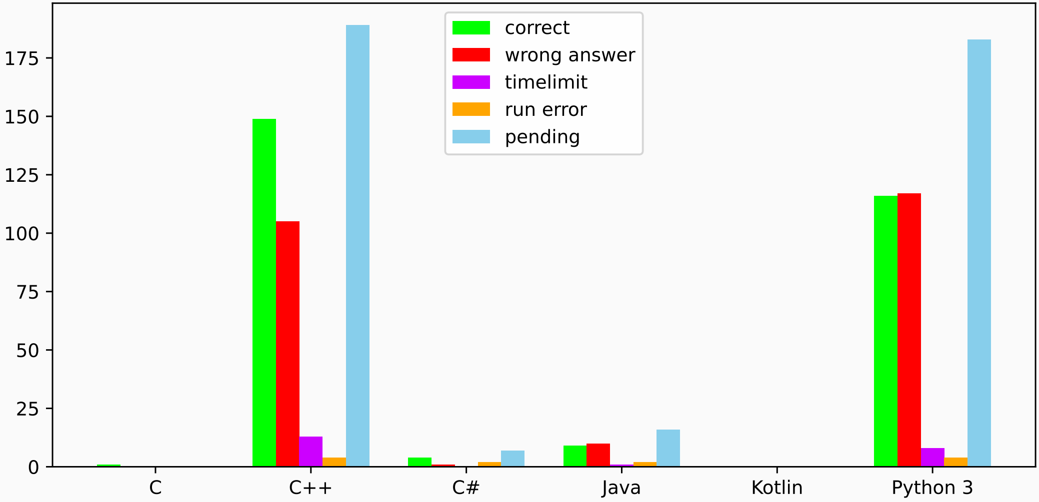
Statistics: 14 submissions, 0 accepted, 12 unknown

Language stats



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- 196 jury solutions (last year: 165)
- The minimum[1] number of lines the jury needed to solve all problems is

$$1+1+7+1+8+2+7+8+15+10+10+14=84$$

On average 7.0 lines per problem, down from 11.9 in BAPC 2022 or 13.9 in preliminaries 2023

[1] We actually had some time to do codegolfing this time, compared to the preliminaries

Thanks to:

The proofreaders

Jaap Eldering
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Michael Vasseur
Nicky Gerritsen (Java Hero 🎈)
Pavel Kunyavskiy (Kotlin Hero 🎈)
Thomas Verwoerd (Kotlin Hero 🎈)

The jury

Gregor Behnke
Ivan Fefer
Jorke de Vlas
Ludo Pulles
Maarten Sijm
Mees de Vries
Mike de Vries
Ragnar Groot Koerkamp
Reinier Schmiermann
Wessel van Woerden

Want to join the jury? Submit to the Call for Problems of BAPC 2024 at:

`https://jury.bapc.eu/`