NWERC 2021 presentation of solutions

November 21, 2021

NWERC 2021 Jury

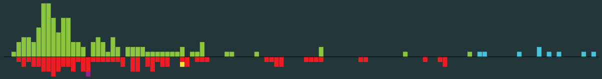
- **Per Austrin**
  KTH Royal Institute of Technology
- **Alexander Dietsch**
  e.solutions
- **Ragnar Groot Koerkamp**
  ETH Zurich
- **Antti Laaksonen**
  CSES
- **Bjarki Ágúst Guðmundsson**
  Google
- **Nils Gustafsson**
  KTH Royal Institute of Technology
- **Timon Knigge**
  ETH Zurich
- **Harry Smit**
  MPIM Bonn
- **Bergur Snorrason**
  University of Iceland
- **Pehr Söderman**
  Kattis
- **Jorke de Vlas**
  Utrecht University
- **Mees de Vries**
  IMC
- **Paul Wild**
  FAU Erlangen-Nürnberg
- **Michael Zündorf**
  Karlsruhe Institute of Technology

Big thanks to our test solvers

- **Bernhard Linn Hilmarsson**
  ETH Zurich
- **Robin Lee**
  Google
- **Ludo Pulles**
  Leiden University
- **Johan Sannemo**
  Kattis
- **Reinier Schmiermann**
  Utrecht University
- **Tobias Werth**
  Google

K: Knitpicking

Problem Author: Pehr Söderman



Problem

Given a drawer full of socks, compute how many you need to pick to be guaranteed to have a pair.

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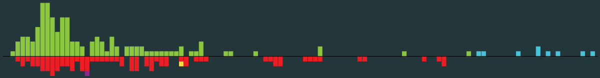

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- Count the number of socks you can pick *without* a pair, then add 1 at the end.

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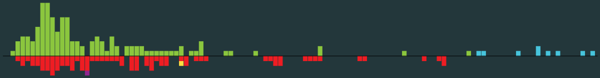

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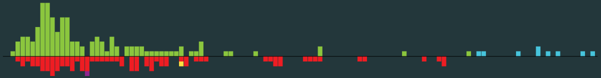

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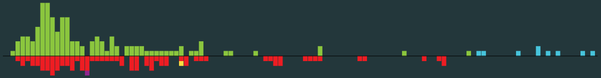

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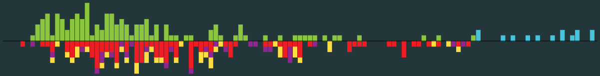
Statistics: 218 submissions, 126 accepted, 9 unknown

A: Access Denied

Problem Author: Pehr Söderman



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Use the timing of a password checker to guess a password.

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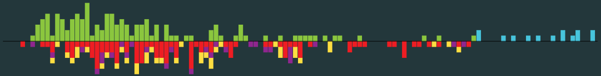

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- First find the length: guess passwords of lengths 1 to 20. The one with the longest timeout gives you the length.

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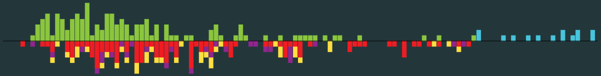

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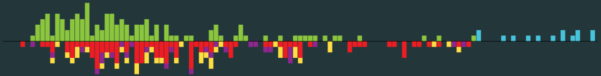

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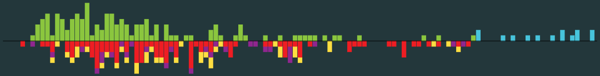

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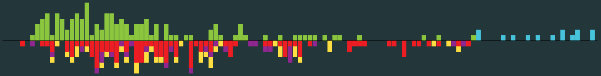

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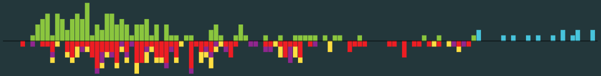

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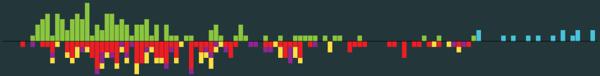

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Statistics: 300 submissions, 118 accepted, 14 unknown

J: Jet Set

Problem Author: Paul Wild

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Given a list of stops on a trip, determine whether it passes through every meridian.

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- Observations:
  - You can ignore the latitudes – they do not matter.

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- Pitfalls: be careful to correctly operate on the circular array.

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Statistics: 342 submissions, 81 accepted, 74 unknown

J: Jet Set

Problem Author: Paul Wild

Edge case

Don't forget the edge case of going around for 359° degrees and then turning around!

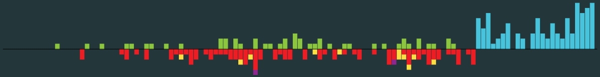
Edge case

```
--- Original
+++ New
@@ @@
         cout <<setprecision(1) << fixed;
         double dres = res/2.0;
         double unfix = dres >= M/2 ? dres -M : dres;
-        cout << unfix << "\n";
+        cout << "no " <<  unfix << "\n";
     }
 }
```

Please read the output section carefully.

D: Dyson Circle

Problem Author: Mees de Vries



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Given some stars on a grid, encircle these with as few other grid points as possible.

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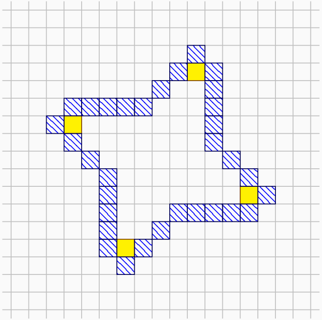
Given some stars on a grid, encircle these with as few other grid points as possible.

Solution

- Let's look at the first sample.



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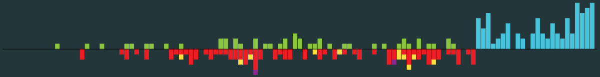

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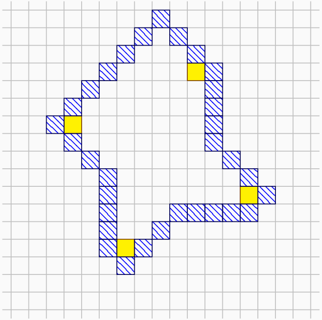
- We might as well remove a "dent" in our Dyson circle.



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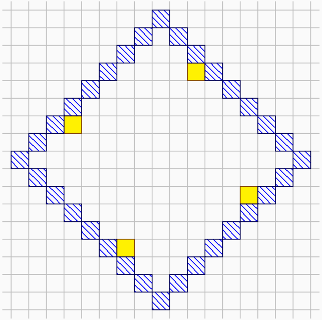

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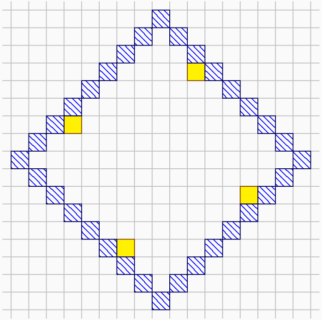
- Let's look at the first sample.
- We might as well remove a "dent" in our Dyson circle.
- In fact, we can do this with all dents.
- In general, a rectangle with diagonal edges is *always* an optimal solution.



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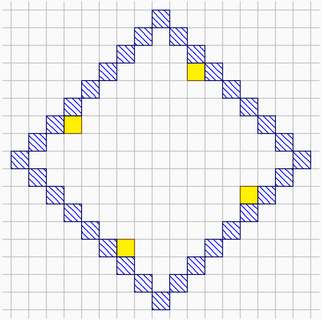
Given some stars on a grid, encircle these with as few other grid points as possible.

Solution

- The only suns that matter are the four suns that touch the edges of the rectangle: the ones that maximize $x + y$, $x - y$, $-x + y$, $-x - y$.



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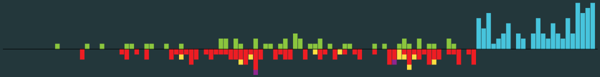

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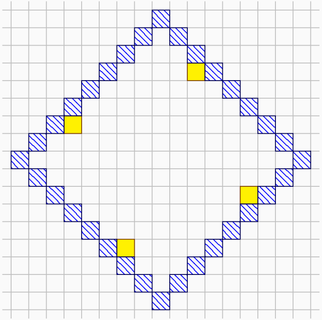
- So the general answer is

$$4 + \max_i(x_i + y_i) + \max_i(x_i - y_i) + \max_i(-x_i + y_i) + \max_i(-x_i - y_i).$$



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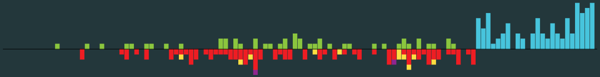

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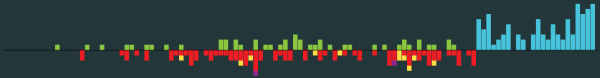

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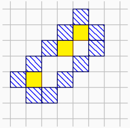
- If all of the suns are on a diagonal, you need one additional square to make the inside a contiguous region.



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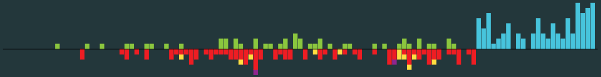

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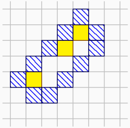
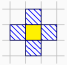
- If all of the suns are on a diagonal, you need one additional square to make the inside a contiguous region.
- However, if there is only one sun you do not need the additional square.





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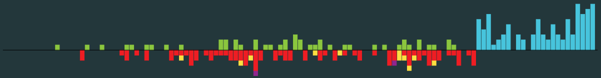

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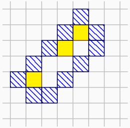

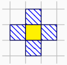

Statistics: 248 submissions, 48 accepted, 99 unknown

G: Glossary Arrangement

Problem Author: Jorke de Vlas

Problem

Given an alphabetical list of n words, split the list up into multiple columns so that the layout is at most w characters wide and the height is minimised.

```
user@pc ~/glossary $ ls
algorithm programming
contest   regional
eindhoven reykjavik
icpc      ru
nwerc
```

```
user@pc ~/glossary $ ls--
algorithm icpc  programming ru
contest   nwerc regional
eindhoven       reykjavik
```

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Solution

- The answer can be found using binary search.

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- Can also speed up DP for an $\mathcal{O}(n \log^2(n))$ solution.

G: Glossary Arrangement

Problem Author: Jorke de Vlas

Solution

- The answer can be found using binary search.
- New problem: Is there a layout of height at most h?
- Given h, solve the new problem using dynamic programming:

$$f(i) = \text{minimal width needed to split the first } i \text{ words into columns}$$

- Number of states is n, and there are at most h transitions from each state.
- Time complexity: $\mathcal{O}(n^2 \log(n))$.
- Can also speed up DP for an $\mathcal{O}(n \log^2(n))$ solution.

Statistics: 102 submissions, 35 accepted, 31 unknown

H: Heating Up

Problem Author: Alexander Dietsch

Problem

Given a pizza with many slices, each having its own spiciness level. Eating a slice with a certain spiciness is only possible if you have enough tolerance, and it increases this tolerance by the spiciness level of the slice.

You are allowed to start at any slice but after every slice, you must continue with one of the neighbouring slices. Which initial minimal tolerance is needed to finish the pizza.

H: Heating Up

Problem Author: Alexander Dietsch

Solution

- Problem can be solved with binary search. (If tolerance x is enough, $x+1$ works as well)

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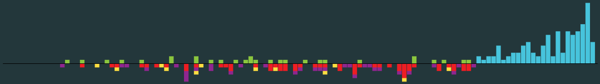
- Problem can be solved with binary search. (If tolerance x is enough, $x + 1$ works as well)
- New problem: Does tolerance x suffice to eat the whole pizza?
- Use a cyclic linked list, each element holds the spiciness level to finish the element and the increase in tolerance it gives. Initially 1 slice $=$ 1 element.

Solution

- Problem can be solved with binary search. (If tolerance x is enough, $x + 1$ works as well)
- New problem: Does tolerance x suffice to eat the whole pizza?
- Use a cyclic linked list, each element holds the spiciness level to finish the element and the increase in tolerance it gives. Initially 1 slice = 1 element.
- Visit all elements; on a visit:



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- Use a cyclic linked list, each element holds the spiciness level to finish the element and the increase in tolerance it gives. Initially 1 slice $=$ 1 element.
- Visit all elements; on a visit:
  - Check if the initial tolerance is high enough to finish the element.

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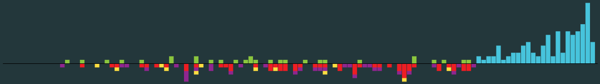
Problem Author: Alexander Dietsch



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- If the linked list can be merged into a single element, the initial tolerance is enough to finish the pizza.

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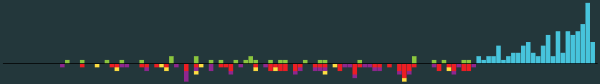

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- If the linked list can be merged into a single element, the initial tolerance is enough to finish the pizza.

Statistics: 252 submissions, 29 accepted, 124 unknown

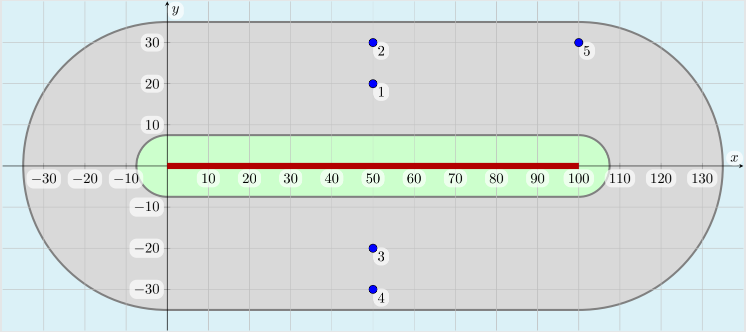
F: Flatland Olympics

Problem Author: Harry Smit

Problem

Given a line segment s and a set of n points $p_1, \ldots, p_n$. Find the number of pairs of points p_i, p_j $(i < j)$ such that both points lie on the same side of s and the line through p_i and p_j intersects s.

Example



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observation

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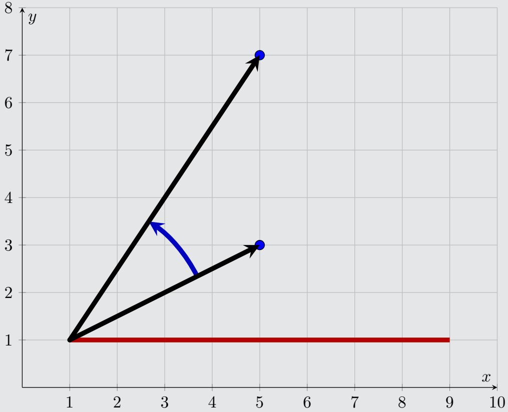

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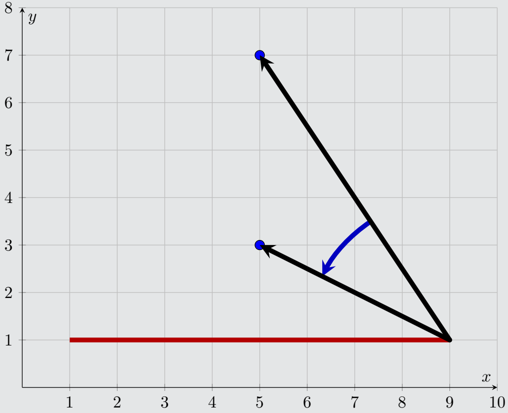

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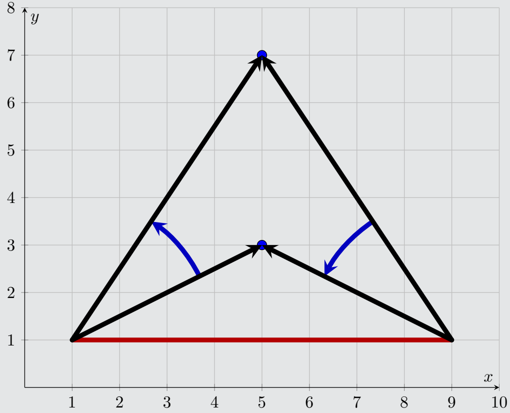

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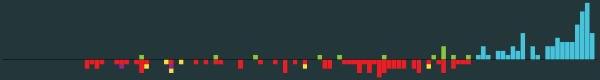

Solution

- Separate the points above and below s in two different sets.

F: Flatland Olympics

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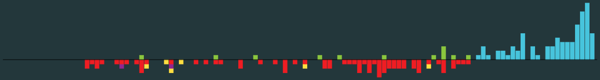

Solution

- Separate the points above and below s in two different sets.
- For each set:
  - Sort the points around the *start* of s.
  - Sort the points around the *end* of s.
  - A pair of points has to be counted if their order in these two sequences differ.

Solution

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- This can be done in $\mathcal{O}(n \log(n))$.

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- We need to find the number of *inversions* between two permutations.
- This can be done in $\mathcal{O}(n \log(n))$.

Gotcha

- Points lying along the line through s.
- Multiple points collinear with the start or the end of s.

Statistics: 179 submissions, 12 accepted, 86 unknown

E: Exchange Students

Problem Author: Nils Gustafsson

Problem

Given two permutations g and h of size $n \leq 300\,000$, turn g into h by swapping pairs of elements with only smaller elements in between them. How many moves are needed and find the first up to 200 000 moves.

E: Exchange Students

Problem Author: Nils Gustafsson

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Given two permutations g and h of size $n \leq 300\,000$, turn g into h by swapping pairs of elements with only smaller elements in between them. How many moves are needed and find the first up to 200 000 moves.

Solution

- Observation: in an optimal solution, you can reorder the swaps to first do all swaps involving the shortest students.

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- When doing swaps involving the shortest students, they always move one step at a time.
- After the shortest students are in place, they do not affect any of the other swaps, and you can remove them from the sequence.

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- Observation: in an optimal solution, you can reorder the swaps to first do all swaps involving the shortest students.
- When doing swaps involving the shortest students, they always move one step at a time.
- After the shortest students are in place, they do not affect any of the other swaps, and you can remove them from the sequence.
- Now your sequence has one fewer height, and you can repeat.

E: Exchange Students

Problem Author: Nils Gustafsson

Solution

- If the shortest students are in locations $a_1, \ldots, a_k$ in g and $b_1, \ldots, b_k$ in h, then it takes

$$\sum_{i=1}^{k} |a_i - b_i|$$

steps to get them into the right location.

Solution

- If the shortest students are in locations $a_1, \ldots, a_k$ in g and $b_1, \ldots, b_k$ in h, then it takes

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  steps to get them into the right location.
- When removing the shortest students, use a Segment or Fenwick tree to keep track of locations of the other students in the new sequence.
- Do the reconstruction while you count the steps, as long as you have not reached the number of steps you have to output.
- Take care to not swap with equal elements. From $1, 1, 2$ to $2, 1, 1$, the first 1 needs to go right, but that is only possible by swapping the 2 to the left.

E: Exchange Students

Problem Author: Nils Gustafsson

Solution

- Challenge: Can you do it in $O(n \lg n + moves)$?

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- Challenge: Can you do it in $O(n \lg n + moves)$?
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Solution

- Challenge: Can you do it in $O(n \lg n + moves)$?
- Challenge: Can you do it in $O(n \lg n + moves \lg n)$, but by processing all elements in random order?

Statistics: 24 submissions, 4 accepted, 13 unknown

Problem Author: Paul Wild

Problem

Given a specific number of each of the letters M, D, C, L, X, V, I, what is the least number of Roman numerals that can be formed while using exactly the required number of each letter?

I: `IXth` Problem

Problem Author: Paul Wild

Problem

Given a specific number of each of the letters `M`, `D`, `C`, `L`, `X`, `V`, `I`, what is the least number of Roman numerals that can be formed while using exactly the required number of each letter?

Insight

We can use binary search on the answer. New subproblem: Given an integer n, can we form at most n numerals using all the tiles?

Solution for subproblem

Start with n empty strings and add the digits in order from M to I.

$$\texttt{M} \times 4 \quad \texttt{D} \times 1 \quad \texttt{C} \times 7 \quad \texttt{L} \times 1 \quad \texttt{X} \times 3 \quad \texttt{V} \times 1 \quad \texttt{I} \times 3$$

1.
2.

Problem Author: Paul Wild

Solution for subproblem

Start with n empty strings and add the digits in order from M to I.

$$\mathtt{M} \times 0 \quad \mathtt{D} \times 1 \quad \mathtt{C} \times 7 \quad \mathtt{L} \times 1 \quad \mathtt{X} \times 3 \quad \mathtt{V} \times 1 \quad \mathtt{I} \times 3$$

1. MMM
2. M

- Distribute M, C, X and I in groups of three, and D, L and V on their own.

Solution for subproblem

Start with n empty strings and add the digits in order from M to I.

$$\mathtt{M} \times 0 \quad \mathtt{D} \times 0 \quad \mathtt{C} \times 7 \quad \mathtt{L} \times 1 \quad \mathtt{X} \times 3 \quad \mathtt{V} \times 1 \quad \mathtt{I} \times 3$$

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1. MMMDCCC
2. MCCC

- Distribute M, C, X and I in groups of three, and D, L and V on their own.

Solution for subproblem

Start with n empty strings and add the digits in order from `M` to `I`.

$$\texttt{M} \times 0 \quad \texttt{D} \times 0 \quad \texttt{C} \times 0 \quad \texttt{L} \times 1 \quad \texttt{X} \times 2 \quad \texttt{V} \times 1 \quad \texttt{I} \times 3$$

1. `MMMDCCCXC`
2. `MCCC`

- Distribute `M`, `C`, `X` and `I` in groups of three, and `D`, `L` and `V` on their own.
- If there is not enough room for all `M`, `C` or `X`, try filling up with copies of `CM`, `XC` or `IX`.

I: `IXth` Problem

Problem Author: Paul Wild

Solution for subproblem

Start with n empty strings and add the digits in order from `M` to `I`.

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2. `MCCCL`

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- Fill up later letters from the first available slot.

Solution for subproblem

Start with n empty strings and add the digits in order from M to I.

$$\texttt{M} \times 0 \quad \texttt{D} \times 0 \quad \texttt{C} \times 0 \quad \texttt{L} \times 0 \quad \texttt{X} \times 0 \quad \texttt{V} \times 1 \quad \texttt{I} \times 3$$

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2. MCCCLXX

- Distribute M, C, X and I in groups of three, and D, L and V on their own.
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Problem Author: Paul Wild

Solution for subproblem

Start with n empty strings and add the digits in order from M to I.

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2. MCCCLXX

- Distribute M, C, X and I in groups of three, and D, L and V on their own.
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Solution for subproblem

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- Fill up later letters from the first available slot.
- If you run out of letters or room at any point, abort.

Solution for subproblem

Start with n empty strings and add the digits in order from M to I.

$$\texttt{M}\times 0 \quad \texttt{D}\times 0 \quad \texttt{C}\times 0 \quad \texttt{L}\times 0 \quad \texttt{X}\times 0 \quad \texttt{V}\times 0 \quad \texttt{I}\times 0$$

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- As the numbers are huge, speed it up by grouping equal numbers together.

I: IXth Problem

Problem Author: Paul Wild

Solution for subproblem

Start with n empty strings and add the digits in order from M to I.

$$\mathtt{M}\times 0 \quad \mathtt{D}\times 0 \quad \mathtt{C}\times 0 \quad \mathtt{L}\times 0 \quad \mathtt{X}\times 0 \quad \mathtt{V}\times 0 \quad \mathtt{I}\times 0$$

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Statistics: 34 submissions, 2 accepted, 20 unknown

B: Boredom Buster

Problem Author: Nils Gustafsson

Problem

Play a single player version of the game Memory (aka Concentration), where the cards are randomly shuffled before and after reveal.

Problem Author: Nils Gustafsson

Problem

Play a single player version of the game Memory (aka Concentration), where the cards are randomly shuffled before and after reveal.

Solution

- **First attempt**: Revealing the cards with indices i and j will give you the card numbers x and y. If you now query (j, k) and you get result (x, z) for some different z, you can deduce that $c_i = y$.
- Repeating this logic $n - 1$ times, $n - 2$ cards will be known. We still have to take care of the last two, but this is too many queries.
- **Insight**: We have to exploit the fact that there are many duplicates in the deck.

B: Boredom Buster

Problem Author: Nils Gustafsson

Solution

- **Attempt 2**: Query for $(1,2),(3,4),(5,6),\cdots$. This gives you $\frac{n}{2}$ tuples on the form (i,j,x,y) meaning that the cards on positions i and j have values x, y.

B: Boredom Buster

Problem Author: Nils Gustafsson

Solution

- **Attempt 2**: Query for $(1, 2), (3, 4), (5, 6), \cdots$. This gives you $\frac{n}{2}$ tuples on the form (i, j, x, y) meaning that the cards on positions i and j have values x, y.
- Take two tuples on the form (i_1, j_1, x, y), (i_2, j_2, y, z) and query (i_1, i_2).

B: Boredom Buster

Problem Author: Nils Gustafsson

Solution

- **Attempt 2**: Query for $(1,2),(3,4),(5,6),\cdots$. This gives you $\frac{n}{2}$ tuples on the form (i,j,x,y) meaning that the cards on positions i and j have values x, y.
- Take two tuples on the form (i_1,j_1,x,y), (i_2,j_2,y,z) and query (i_1,i_2).
  - With 75% probability the answer will be different numbers (e.g. (x,z)). This will give you two card positions and creates another tuple (i_1,i_2,x,z).

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- Take two tuples on the form (i_1, j_1, x, y), (i_2, j_2, y, z) and query (i_1, i_2).
  - With 75% probability the answer will be different numbers (e.g. (x, z)). This will give you two card positions and creates another tuple (i_1, i_2, x, z).
  - With 25% probability the answer will be (y, y) which gives you all four cards.

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- Take two tuples on the form (i_1,j_1,x,y), (i_2,j_2,y,z) and query (i_1,i_2).
  - With 75% probability the answer will be different numbers (e.g. (x,z)). This will give you two card positions and creates another tuple (i_1,i_2,x,z).
  - With 25% probability the answer will be (y,y) which gives you all four cards.
- By naively pairing up the tuples to get these collisions and executing the above strategy, you will solve the problem with around $\frac{15}{16}n$ queries. But this is still not enough!

Solution

- How to cause many collisions using the idea on the previous slide?

B: Boredom Buster

Problem Author: Nils Gustafsson

Solution

- How to cause many collisions using the idea on the previous slide?
- Let's model the problem as a graph with $\frac{n}{2}$ vertices, one for each card number.

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- Let's model the problem as a graph with $\frac{n}{2}$ vertices, one for each card number.
- For every tuple (i, j, x, y) add an edge between x and y. This gives components which are either cycles or paths.

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Problem Author: Nils Gustafsson

Solution

- How to cause many collisions using the idea on the previous slide?
- Let's model the problem as a graph with $\frac{n}{2}$ vertices, one for each card number.
- For every tuple (i, j, x, y) add an edge between x and y. This gives components which are either cycles or paths.
- Querying for pairs of adjacent edges has a chance of eliminating both edges.

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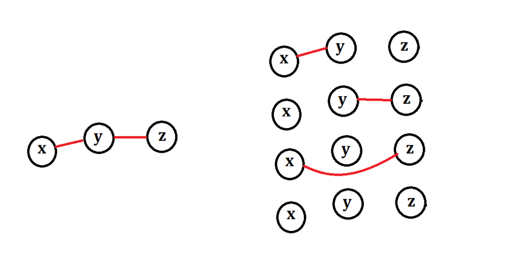
Problem Author: Nils Gustafsson

Solution

- How to cause many collisions using the idea on the previous slide?
- Let's model the problem as a graph with $\frac{n}{2}$ vertices, one for each card number.
- For every tuple (i, j, x, y) add an edge between x and y. This gives components which are either cycles or paths.
- Querying for pairs of adjacent edges has a chance of eliminating both edges.



B: Boredom Buster

Problem Author: Nils Gustafsson

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Statistics: 23 submissions, 1 accepted, 8 unknown

L: Lucky Shirt

Problem Author: Ragnar Groot Koerkamp

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Given is a list of n shirts. We choose k integers $l_1, \ldots, l_k$ uniformly at random and then randomly permute the first l_j shirts for $j \in \{1, \ldots, k\}$. What is the expected position of the shirt that started at position i (1-based)?

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- However, p_a does not have a nice formula.

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  - The (expected) position of the shirt is $(M+1)/2$.

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$$i \cdot \mathbb{P}(M < i) + \sum_{a=i}^{n} \frac{a+1}{2} \cdot \mathbb{P}(M = a).$$

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$$\mathbb{P}(M < i) = \left(\frac{i-1}{n}\right)^k, \text{ and}$$

$$\mathbb{P}(M = a) = \mathbb{P}(M < a+1) - \mathbb{P}(M < a) = \left(\frac{a}{n}\right)^k - \left(\frac{a-1}{n}\right)^k.$$

Statistics: 30 submissions, 1 accepted, 25 unknown

C: Cutting Edge

Problem Author: Paul Wild

Problem

Given a desired volume $v/6$, find a set of integer-valued points whose convex hull has this volume.

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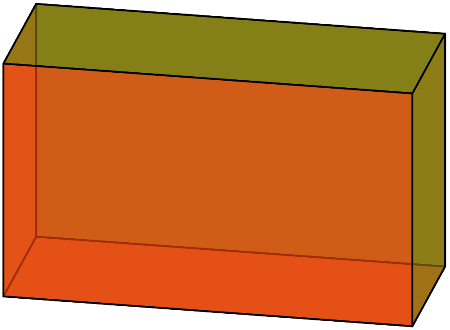
General idea

- Start with a cuboid and cut away tetrahedra from four of the corners.



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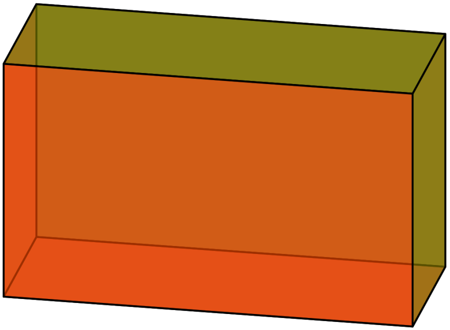

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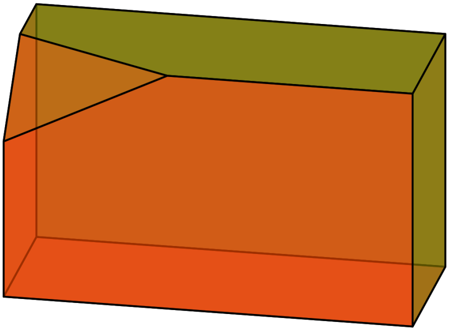

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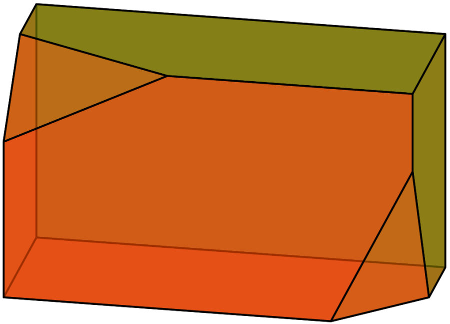

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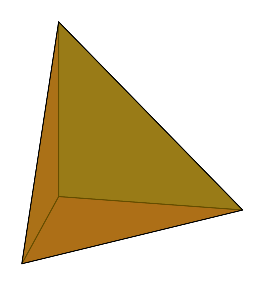
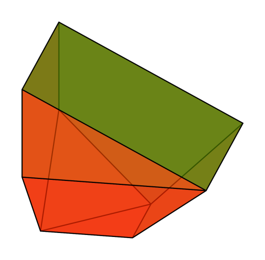
- If $c \leq 5$, then so are a and b, which implies that $|S|$ is small (at most 31).
- Brute force all combinations to check if r can be written as a sum of four elements in S.

Leftover cases

This solves all cases except for two:

- The case where $a = b = c = 1$ and $v = 1$. This is the first sample.





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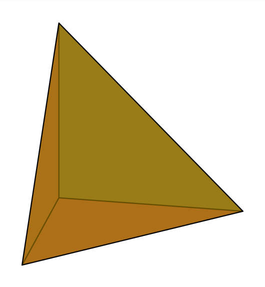
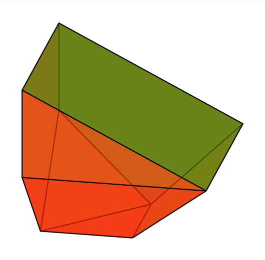
- The case where $a = b = c = 1$ and $v = 1$. This is the first sample.
- The case where $a = b = c = 2$ and $v = 25$. Then $r = 23$. This is the third sample.





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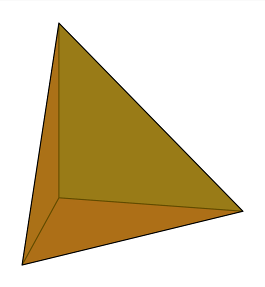

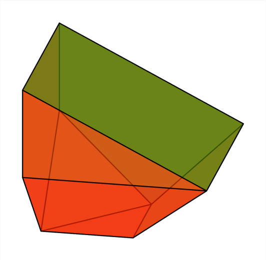

Statistics: 4 submissions, 0 accepted, 4 unknown

Random facts

Jury work

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[1] *After* codegolfing

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- 248 jury solutions (last year: 232)
- The minimum[1] number of lines the jury needed to solve all problems is

$$10 + 114 + 27 + 5 + 64 + 51 + 42 + 32 + 43 + 23 + 10 + 6 = 427$$

  On average 35.5 lines per problem, up from 9.6 in the BAPC

[1] *After* codegolfing