BAPC 2021 Preliminaries Solutions presentation

October 9, 2021

BAPC 2021 Preliminaries



Problem Author: Mees de Vries



Problem: Given a word, decide if it is *Dickensian* (i.e., typeable alternatingly with left and right hand)

Statistics: 39 submissions, 33 accepted, 5 unknown

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Solution: Check for every letter whether it is typeable with left or right



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- Problem: Given a word, decide if it is *Dickensian* (i.e., typeable alternatingly with left and right hand)
- Solution: Check for every letter whether it is typeable with left or right
- Check if the resulting list is alternating
 - Note that you can start with either left or right



Problem Author: Robin Lee



 Problem: Keep Kevin busy by asking him to take items from his long fridge. Ask for as little items as possible.

Statistics: 68 submissions, 22 accepted, 26 unknown

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- Problem: Keep Kevin busy by asking him to take items from his long fridge. Ask for as little items as possible.
- Solution: Keep asking items from the very back of the fridge
 - Maintaining a rotating list of items will do
 - For the last item, pick one from the middle, based on the number of seconds left





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 - Maintaining a rotating list of items will do
 - For the last item, pick one from the middle, based on the number of seconds left
- Optimization: You don't need to maintain a list if you do some math





Problem: What is the minimum circumference of the table such that everyone can comply with their social distancing requirement?

Statistics: 65 submissions, 21 accepted, 16 unknown

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- Problem: What is the minimum circumference of the table such that everyone can comply with their social distancing requirement?
- Solution: Order guests by their required distance,
 - then you can sum up their distances to get the table circumference.
 - The guest requiring the smallest distance is always satisfied on both sides, so this guest should not be counted.
 - The guest requiring the largest distance, requires this distance on both sides, so this guest should be counted twice.
 - All other guests are automatically satisfied on the side where somebody with lesser requirements is sitting, so they only need to be counted once.





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• Final answer:
$$\sum_i d_i + \max_i(d_i) - \min_i(d_i)$$





Problem Author: Abe Wits

Problem: Print a histogram with the given data.





- **Problem:** Print a histogram with the given data.
- Solution: First count the size for each bin, then print the histogram.
 - Make sure to calculate the height of the histogram beforehand



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 - Ice-thickness can't be negative.
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- For each person binary search how many days have the required thickness $[\mathcal{O}(k \log(n))].$
- Alternative: store the number of days for each ice-thickness $\leq 10^6$, and accumulate once $[\mathcal{O}(k+n)]$.

Problem Author: Ruben Brokkelkamp

Problem: Given a graph, nodes s and t, a number of candies c and for each edge e an integer p_e denoting what percentage of the candies you are carrying you have to pay to use the edge (rounded up).

Statistics: 57 submissions, 7 accepted, 34 unknown BAPC 2021 Preliminaries

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- Sample showed that computing path with lowest summed taxed percentage is not always best: (1 0.25)(1 0.1) = 0.675 > 0.672 = (1 0.04)(1 0.3).



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So, cannot do a 'normal' additive dijkstra with tax percentages to find best path.
Solution: Tweak dijkstra a bit. Instead of initializing every node to ∞ and lowering it everytime you find a shorter path. Initialize everything to 0 and raise it when you find a path where you hold on to more candies.



Problem: Given a file movement $s_1/s_2/.../s_n \rightarrow t_1/t_2/.../t_m$ find the shortest move description, assuming that the s_i are distinct and the t_j are distinct.

G: Git mv Problem Author: Ragnar Groot Koerkamp

- **Problem:** Given a file movement $s_1/s_2/.../s_n \rightarrow t_1/t_2/.../t_m$ find the shortest move description, assuming that the s_i are distinct and the t_i are distinct.
- **Solution:** Greedy, i.e. find smallest *i* such that $s_i \neq t_i$ and smallest *j* s.t. $s_{n-j} \neq t_{m-j}$. Output:

$$s_1/s_2/\ldots/s_{i-1}/\{s_i/\ldots/s_{n-j}\Longrightarrow t_i/\ldots/t_{m-j}\}/s_{n-j+1}/\ldots/s_n.$$



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- Naive solution: For each pair try whether x_i divides x_j . $\mathcal{O}(n^2)$ is too slow.

A: Almost Always

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- Naive solution: For each pair try whether x_i divides x_j . $\mathcal{O}(n^2)$ is too slow.
- Early break: stop as soon as you find a good pair. O(a/ln(a)) ≈ O(10⁸) expected steps is likely still too slow on the worst of the 100 test cases.



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- Single pass solution: Keep the index of the smallest number seen so far, and check whether it divides the current number.



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- Observation: small numbers are more likely to divide another number.
- Greedy solution: Sort the input before doing the brute force with early break.
- Single pass solution: Keep the index of the smallest number seen so far, and check whether it divides the current number.
- Analysis:

The expected value of the smallest integer is $s \approx a/n = 4000$, so likely below 8000.

The probability that none of the $n = 5 \cdot 10^5$ integers is a multiple of $s \le 8000$ is less than 10^{-27} .

If s does not work, we just try the next smallest integer. (But the probability of needing this is 10^{-5} , so only trying the smallest one is sufficient.)







Bonus solution: Use the birthday paradox.
 The probability that all numbers in the list are distinct is only 7 \cdot 10⁻²⁸, so we can

just find and print the indices of two equal numbers.



Problem: count the number of removed vertices

Statistics: 44 submissions, 1 accepted, 42 unknown

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- Challenge: the number of vertices is at most 2^{10⁶}

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- Observation: the number of removed vertices is 2^{age} unless some subtree is already removed
- Therefore: remember for every vertex how many children are removed and propagate this value to ancestors
- Solution: when removing vertex v with age i, return:

$$2^i - removed[v] \mod 10^9 + 7$$

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Problem Author: Pim Spelier

Problem: Given *n* integers between 1 and 1000, find which of them can be removed such that the remainder can be partitioned into two sets of equal sum.

Statistics: 27 submissions, 3 accepted, 23 unknown

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- Define $s := n \cdot w$ to be the sum of the integers.

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- Define $s := n \cdot w$ to be the sum of the integers.
- $\mathcal{O}(n^2 \cdot s) = \mathcal{O}(n^3 \cdot w)$ solution: For each mole run a $\mathcal{O}(n \cdot s)$ knapsack to check if a partitioning is possible.

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This is usually too slow, unless using bitsets in C++.

- $\mathcal{O}(n^2 \cdot w)$ solution:
 - For each prefix of moles, compute all possible weights of a subset in $\mathcal{O}(n \cdot s)$.
 - For each suffix of moles, compute all possible weights of a subset in $\mathcal{O}(n \cdot s)$.
 - Mole *i* can be left out if it is possible to make a subset of size *l* with the moles before *i*, and a subset of size $(s w_i)/2 l$ of the moles after *i*, for some *l*.

Statistics: 27 submissions, 3 accepted, 23 unknown

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Problem Author: Reinier Schmiermann

• **Problem:** Two players take turns claiming vertices of a graph and get a point every time they claim a vertex adjacent to an enemy vertex. Who wins?

Statistics: 20 submissions, 0 accepted, 20 unknown

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- Observation: The exact scores of the players do not matter, only their difference does.
- Idea: Use DP to find the score difference in remainder of the game, for every game state, assuming optimal play.
- Issue: $\mathcal{O}(3^n/\sqrt{n})$ game states, too many!

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- Results in the same score difference as the original game.
- Vertices are either "claimed" or "unclaimed", so only $\mathcal{O}(2^n)$ game states.
- Using a subset DP: $\mathcal{O}(2^n \cdot n^2)$ time needed.

Language stats



Some stats

- 347 commits (last year: 527)
- 437 secret testcases (last year: 360)
- 175 jury solutions (last year: 221)
- The minimum number of lines the jury needed to solve all problems is

2 + 2 + 10 + 2 + 20 + 3 + 4 + 4 + 9 + 16 + 10 = 82

On average 7.5 lines per problem, down from 13.9 last year

Some tips

- Read the output specification carefully!
- Don't forget to remove debug prints!
- When integers get large, use 64-bit!
- Do not do string concatenation with "+" in a loop!
- Calling functions is more expensive than you might think!

Thanks to the Proofreaders!

Abe Wits Nicky Gerritsen Jaap Eldering Mark van Helvoort Kevin Verbeek

The Jury

Boas Kluiving Erik Baalhuis Freek Henstra Harry Smit Joey Haas Jorke de Vlas Ludo Pulles Maarten Sijm Mees de Vries Ragnar Groot Koerkamp Reinier Schmiermann Robin Lee Ruben Brokkelkamp Timon Knigge Wessel van Woerden