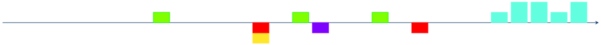
BAPC 2019

Solutions presentation



A: Appeal to the Audience

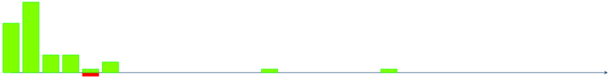
Problem Author: Ragnar Groot Koerkamp

- Place teams in a tournament bracket such that the amount of skill on display to the audience is maximized.
- The best team must play the most games, 2nd the 2nd-most, etc.
- For each node, find the longest path from each child to a leaf recursively.
- Extend the longest of these path to the current node.
- Sort both the list of path lengths and the skill levels.
- Match these lists one to one for a maximal solution.

Statistics: 15 submissions, 3 accepted, 8 unknown

B: Breaking Branches

Problem Author: Timon Knigge



Given a branch of length n, determine who will win the game if they break it into pieces repeatedly.

- A branch of length n can be cut in exactly $n-1$ places.
- After any cut, the number of remaining possible cuts decreases by exactly one.
- Alice wins when n is even. She can break it at any position.

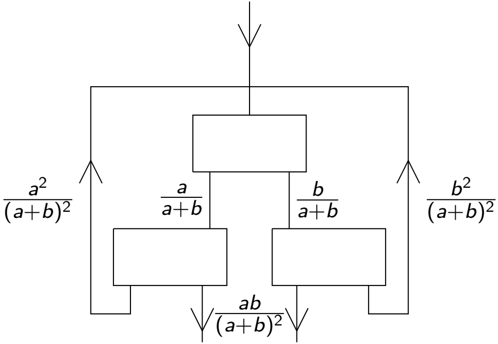
Statistics: 57 submissions, 56 accepted, 0 unknown

C: Conveyor Belts

Problem Author: Daan van Gent

Given splitters that split their input producing an output ratio between their outputs of $(a : b)$, can you build a network of splitters that produces an output ratio $(c : d)$?

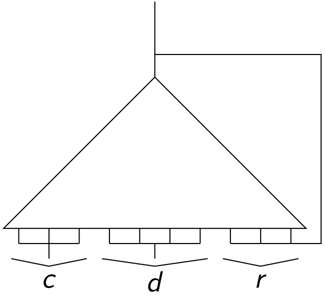
Step 1: Produce a ratio of $(1 : 1)$.



C: Conveyor Belts

Problem Author: Daan van Gent

Step 2: Create a binary tree of depth n of these $(1 : 1)$ splitters, with n such that $c + d \leq 2^n$. Then connect c of the leafs to output 1, d leafs to output 2, and $r = 2^n - (c + d)$ back to the root.



C: Conveyor Belts

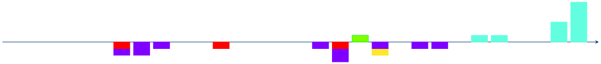
Problem Author: Daan van Gent

Step 3: Remove all $(1:1)$ splitters whose outputs are the same. To not run foul of timelimits, this needs to be done during generation of the tree.

Statistics: 1 submissions, 1 accepted, 0 unknown

D: Deck Randomisation

Problem Author: Ragnar Groot Koerkamp

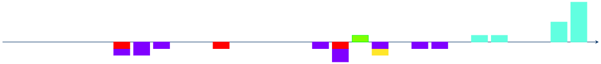


Given two permutations A and B, how often do we need to repeat them one after the other to get back to where we started. Two possibilities:

- Option 1: $(AB)^n = 1$, minimum of $2n$
- Option 2: $(AB)^n A = 1$, minimum of $2n + 1$.

Both take similar strategy.

D: Deck Randomisation



Problem Author: Ragnar Groot Koerkamp

First, calculate the permutation AB, and split it into cycles. Example (notation from problem):

A = 5 1 6 3 2 4

B = 4 6 5 1 3 2

AB = 3 4 2 5 6 1

Then AB has cycle (1 3 2 4 6 5)

For option 1:

- A cycle of length k_i implies $k_i | n$.
- Hence $n = lcm(k_i)$.
- Example: 1 cycle of length 6, so $n = 6$, giving 12 shuffles.

For option 2:

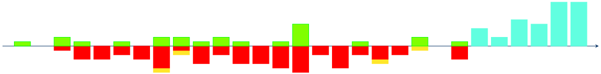
- Per cycle of length k_i , check if exists z such that $(AB)^z = A$. Then $n = -z(\text{mod} k_i)$.
  - Example: $(AB)^4 = 516324$, so $n = -4(\text{mod}6)$.
- Option 2 is not applicable if such z does not exist.
- Then reconstruct n using the Chinese Remainder Theorem.
  - Example: $n = 2$, giving 5 shuffles.

Minimum of the two is answer.

Statistics: 26 submissions, 1 accepted, 11 unknown

E: Efficient Exchange

Problem Author: Raymond van Bommel

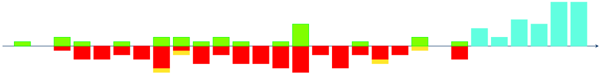


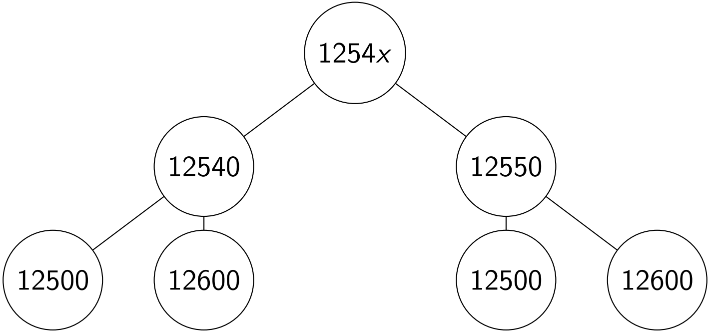
Compute the minimal number of coin exchanges needed to pay n.

- Note that you never need to exchange more than 9 coins of value 10^k, because for 10 exchanges we just use a single 10^{k+1} coin.
- Consider $1254x$. If we can do 12540 in a exchanges and 12550 in b, then we can do
$$\min\{a + x, b + 10 - x\}.$$
- Solving recursively is costs 2^{1000} calls.

E: Efficient Exchange

Problem Author: Raymond van Bommel



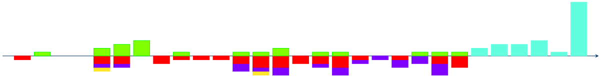


There are only two distinct nodes per layer. So only 1000 calls needed.

Statistics: 124 submissions, 23 accepted, 37 unknown

F: Find my Family

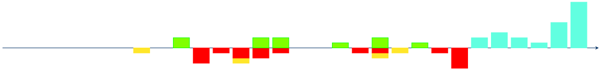
Problem Author: Bjarki Ágúst Guðmundsson



- Given a family picture, find if there are three people of relative height 2, 1, and 3, in this order.
- For each position, find the largest value on the right of it.
- Go from left to right and keep a set of all values seen so far.
- For each value, find the smallest element of the set that is larger.
- A 213 ordering exists if this smallest larger element on the left is smaller than the largest element on the right.

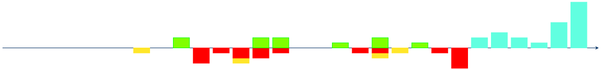
Statistics: 100 submissions, 20 accepted, 27 unknown

G: Gluttonous Goop



Problem Author: Mees de Vries

- Your fungus is growing. How many squares does it occupy after k steps?
- Obvious solution: flood fill. But k can be 10^6: too slow.
- We need to be smarter. Let's look at an example.



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Problem Author: Mees de Vries

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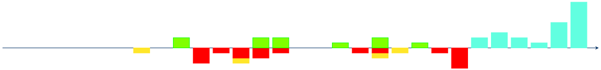

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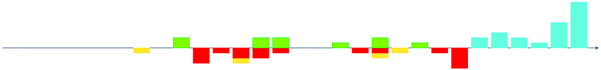

Problem Author: Mees de Vries

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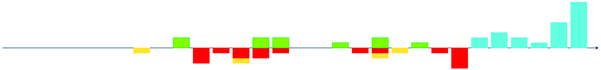

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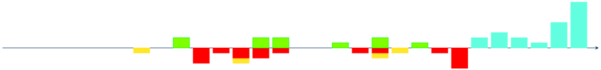

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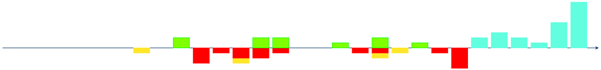

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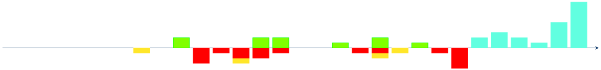

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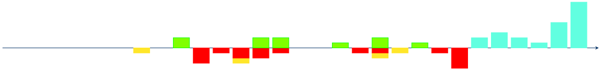

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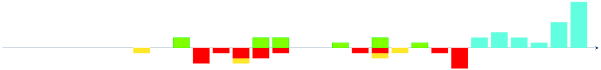
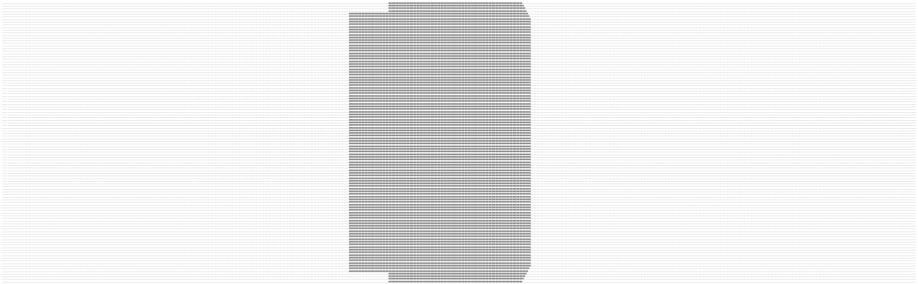


G: Gluttonous Goop

Problem Author: Mees de Vries

Eventually everything turns into: a big rectangle with weird corners.



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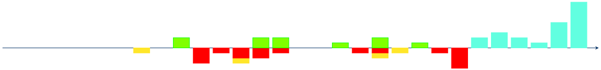


Problem Author: Mees de Vries

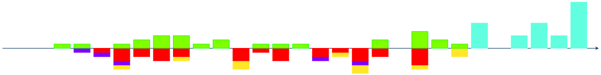
Solution:

- Simulate for 20 steps, then compute the height/width of the bounding rectangle.
- Count how many squares in the rectangle are missing.
- Find the height $\times$ width for the final bounding rectangle. Subtract corner squares.

Statistics: 52 submissions, 10 accepted, 22 unknown

H: Historic Exhibition

Problem Author: Bruno Ploumhans

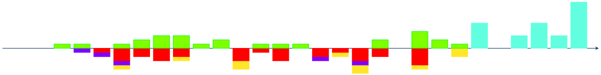


Given k vases, and p pedestals, place every vase on a pedestal of the right size.
Observations:

- If a pedestal has only one size, match those to vases of that size.
- Then you only have pedestals $(1, 2), (2, 3), (3, 4), \ldots$.
- Match all 1-vases to $(1, 2)$-pedestals.
- Use leftover $(1, 2)$-pedestals for 2-vases.
- Then you only have pedestals $(2, 3), (3, 4), (4, 5), \ldots$.
- ... and repeat.

In other words: go greedily from left to right.

H: Historic Exhibition



Problem Author: Bruno Ploumhans

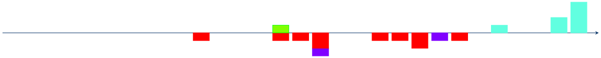
Common pitfalls:

- Output `impossible` once the smallest vase does not fit the smallest available pedestal. (There might still be larger pedestals!)
- Flow algorithm: too slow.

Statistics: 100 submissions, 26 accepted, 29 unknown

I: Inquiry II

Problem Author: Timon Knigge

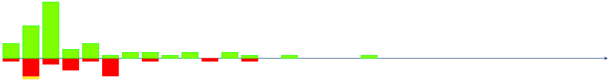


- Given a graph, find a maximum independent set.
- Normally this is NP complete, but ... this graph is very close to a tree!
- We can solve this problem in linear time for a tree.
- Solution:
  - Find the k additional edges, at most 16.
  - For each additional edge, at least one end point is not in the independent set. Brute force all 2^k options.
- Total runtime $O(2^k \cdot n)$.

Statistics: 20 submissions, 1 accepted, 7 unknown

J: Jazz it Up!

Problem Author: Ragnar Groot Koerkamp



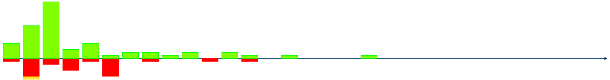
Given a squarefree number n, find an $1 < m < n$ such that $n \times m$ is squarefree.

- Squarefree test: trial division by all squares smaller than n.
- To find m: try all options starting at 2. This is fast since you'll allways find a solution among the first 13 primes.
- Do **NOT** print $n - 1$. It fails for $k^2 + 1$.

Statistics: 80 submissions, 56 accepted, 0 unknown

J: Jazz it Up!

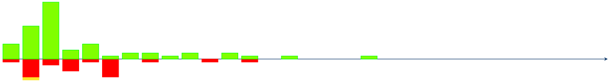
Problem Author: Ragnar Groot Koerkamp



```
static int primes[]={2,3,5,7,11,13,17,19,23,29,31,37, ..., 99991}

int main() {
    int n;
    cin >> n;
    for (int i = 0; i < (int)sizeof(primes) / (int)sizeof(primes[0]);
        i++) {
        if (n % primes[i] != 0) {
            cout << primes[i] << endl;
            break;
        }
    }
}
```

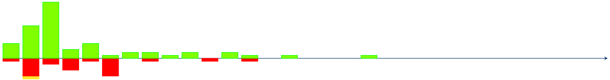
J: Jazz it Up!



Problem Author: Ragnar Groot Koerkamp

```
t = int(input())
P = [2,3,5,7,11,13,17,19,23,29]
for i in P:
    if t%i!=0:
        print(i)
        break
```

J: Jazz it Up!



Problem Author: Ragnar Groot Koerkamp

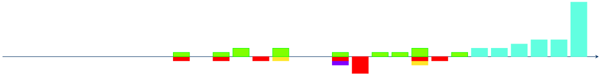
```python
# Get input
n = int(raw_input())

# Find a number m such that m is relatively prime to n

# Check if any of these numbers divides n
if n%2 == 0:
    if n%3 == 0:
        if n%5 == 0:
            if n%7 == 0:
                if n%11 == 0:
                    if n%13 == 0:
                        if n%17 == 0:
                            print 19
                        else:
                            print 17
                    else:
                        print 13
                else:
                    print 11
            else:
                print 7
        else:
            print 5
    else:
        print 3
```

K: Keep Him Inside

Problem Author: Timon Knigge



- The problem was to find a weighted average of the vertices of some convex polygon that equals the given point P.
- Choose one of the vertices of the polygon as a “base point”, translate so the base point is the origin and triangulate the polygon by drawing lines from the base point.
- Find the triangle in which the prisoner P lies (e.g. by calculating angles from the base point). Call the vectors from the base point to two other vertices of the triangle $\mathbf{v}_1$ and $\mathbf{v}_2$
- The vector P can now be decomposed into $a \times \mathbf{v}_1 + b \times \mathbf{v}_2$ by projecting (calculate some inner products).
- The weights are $1 - a - b$, a and b for the three points of the triangle and 0 otherwise.

Statistics: 53 submissions, 12 accepted, 28 unknown

L: Lucky Draw

Problem Author: Raymond van Bommel and Mees de Vries

At the casino, n players start with k lives each. Each round, each one loses a life with probability $1 - p$. You win if you are the only one remaining. What is the probability of a draw?

$$\begin{aligned}
\mathbb{P}(\text{draw}) &= 1 - \mathbb{P}(\text{someone wins}) \\
&= 1 - n \times \mathbb{P}(\text{player 1 wins}) \\
&= 1 - n \times \sum_{i=1}^{\infty} \mathbb{P}(\text{player 1 dies round } i\text{, other player die before round } i) \\
&= 1 - n \times \sum_{i=1}^{\infty} \mathbb{P}(\text{player 1 dies round } i) \times \mathbb{P}(\text{Player 1 dies before round } i)^{n-1}.
\end{aligned}$$

For large i, $\mathbb{P}(\text{player 1 dies round } i)$ is very small. So compute only for i up to $M = 1000$ (or more).

L: Lucky Draw

Problem Author: Raymond van Bommel and Mees de Vries

Two options:

1. Mathematically:

$$\mathbb{P}(\text{Player 1 dies round } i) = \binom{i-1}{k-1} p^{i-k}(1-p)^k.$$

   This gives an $\mathcal{O}(M)$ algorithm.

2. With dynamic programming: let

$$DP[r][l] = \mathbb{P}(\text{Player 1 has } l \text{ lives in round } r.).$$

   Then:

$$DP[r][l] = pDP[r-1][l] + (1-p)DP[r-1][l+1]$$

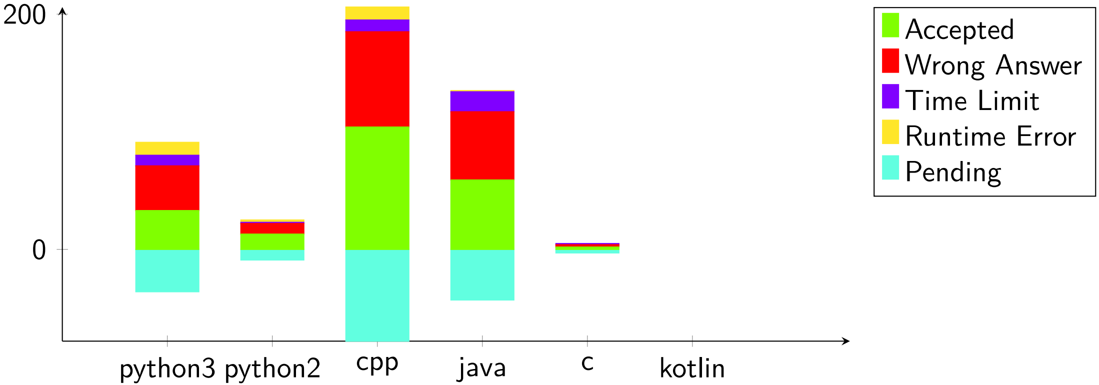
   (plus correct edge conditions). This gives an $\mathcal{O}(Mk)$ algorithm.

L: Lucky Draw

Problem Author: Raymond van Bommel and Mees de Vries

Statistics: 3 submissions, 2 accepted, 0 unknown

Language stats



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