The 2009 ACM North Western European Regional Contest Friedrich-Alexander-University, Nuremberg, Germany

NWERC Jury

November 8, 2009

jury sample solutions

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An Industrial Spy

- generate all possible numbers
- use backtracking or `next_permutation`
- test primality by trial division or sieve of eratosthenes

Common Subexpression Elimination

- parse expression (recursive descent)
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- time $O(n \cdot \log n)$ and space $O(n)$

Divisible Subsequences

- naive $O(n^2)$ solution times out
- instead, compute partial sums modulo d
- if two partial sums have the same remainder, their difference is divisible by d
- for each remainder, save the number of corresponding partial sums
- time $O(n + d)$, space $O(d)$

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- For each depth, iterate over the line segments until you pass the fraction f.
- Then rotate and scale your basis and proceed with the next depth.
- Note: using complex numbers (e.g. C++'s `complex<double>`) is convenient to represent coordinates for scaling/rotating.
- time $O(n \cdot d)$

Mountain Road

- Sort incoming cars into two lists: left-goers and right-goers
- dynamic programming: find the optimal time when a left-goers and b right-goers have passed and the last car was of type A or B
- try to send $1, 2, 3, \ldots$ cars at a time
- time $O(n^3)$, space $O(n^2)$

Moving to Nuremberg

- For each v, want sum of distances $D(v)$ from v to every other node (weighted by their frequencies)
- Easy to compute contribution to $D(v)$ from nodes in subtree rooted at v
- Find formula for remaining part in terms of $D(\text{parent})$
- Propagate down from the root
- greedy optimisation of convex function
- time $O(n)$

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- each connected component must contain at most one cycle
- one tree component, additional components with exactly one cycle

Room assignments

- Distinguish 3 cases:
  1. one component with more than cycle: impossible
  2. only one component: choose the two nodes with highest rating
  3. more components: choose room with highest rating from tree component
     second room either from tree (same rating) or from cyclic component
- time $O(n)$, space $O(n)$

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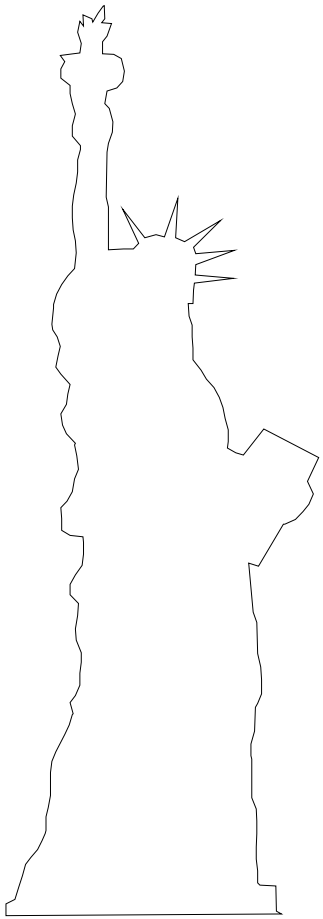
- time $O(n)$ using precomputation
- How to represent a hexagonal lattice?
- How to simulate the spiral?
- Good exercise for solving ad hoc problems and finding easy to code representations.

Simple Polygon



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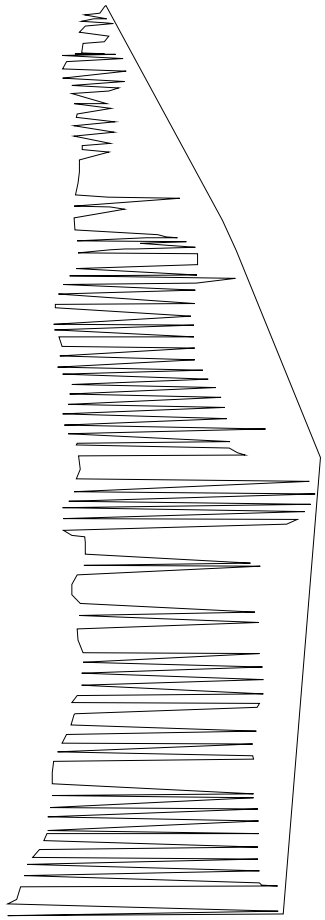

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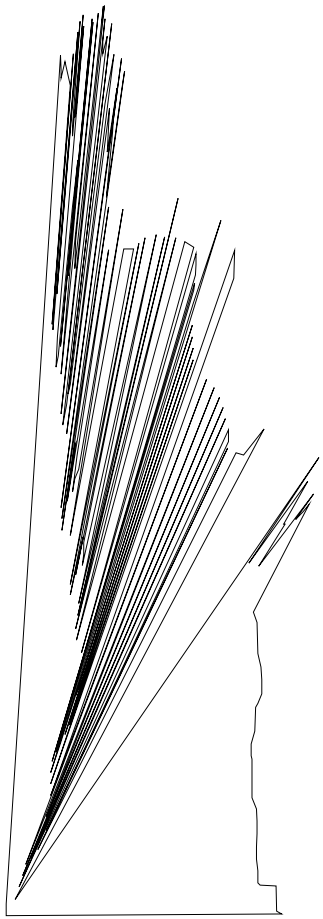

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This gives an $\mathcal{O}(n^4)$ algorithm.

Award Ceremony